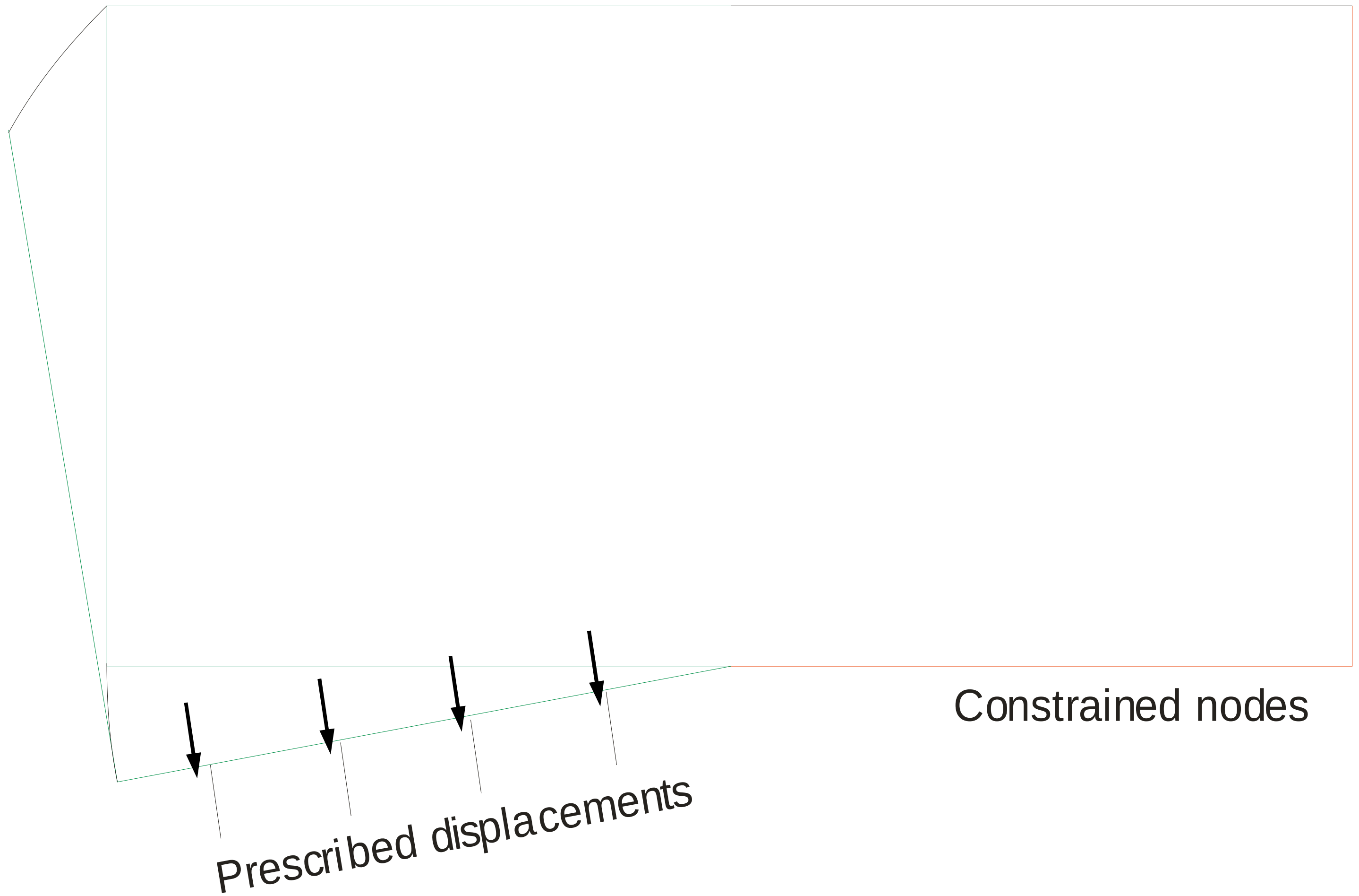
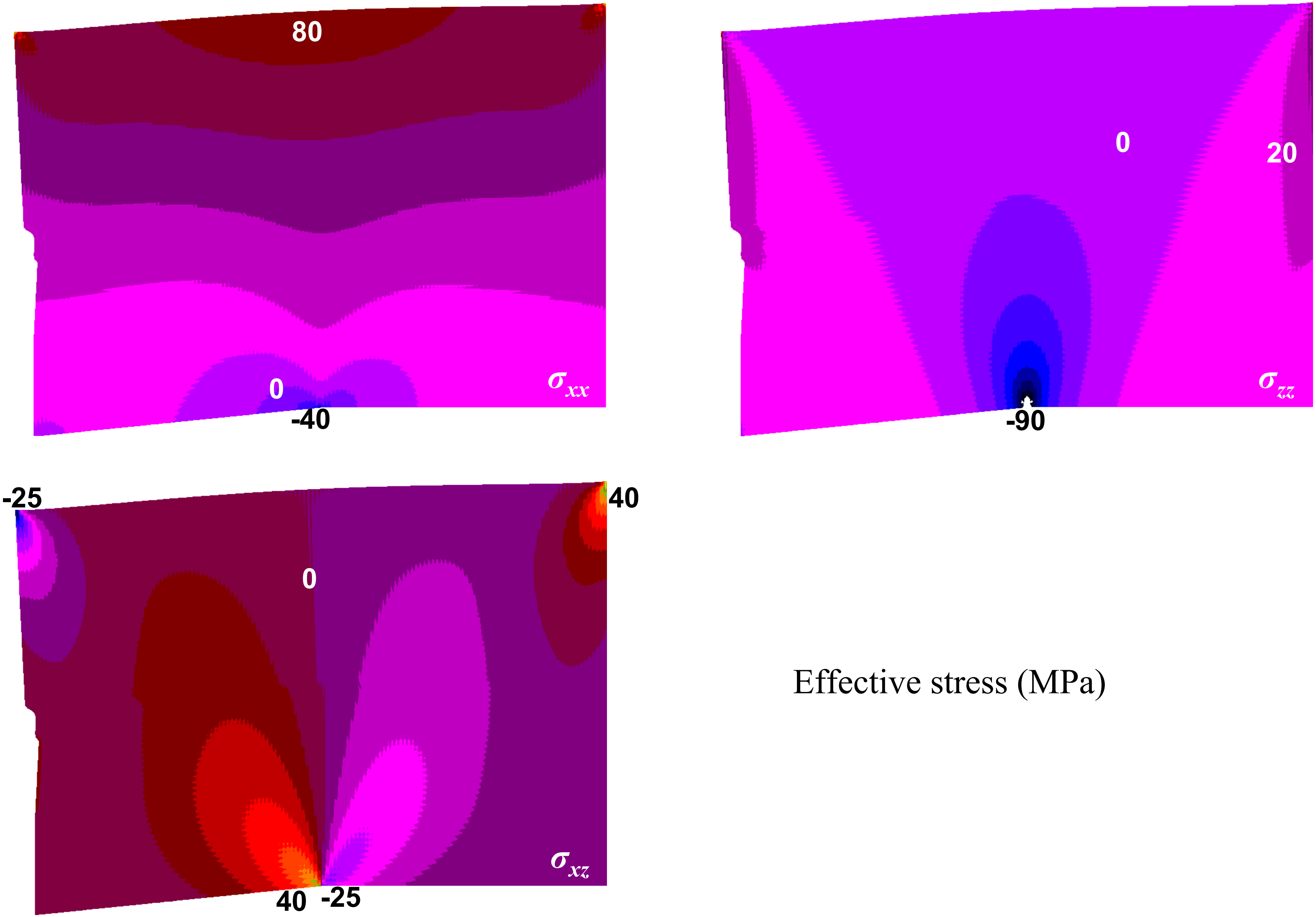
Rock Mechanics Seminar Series 2010

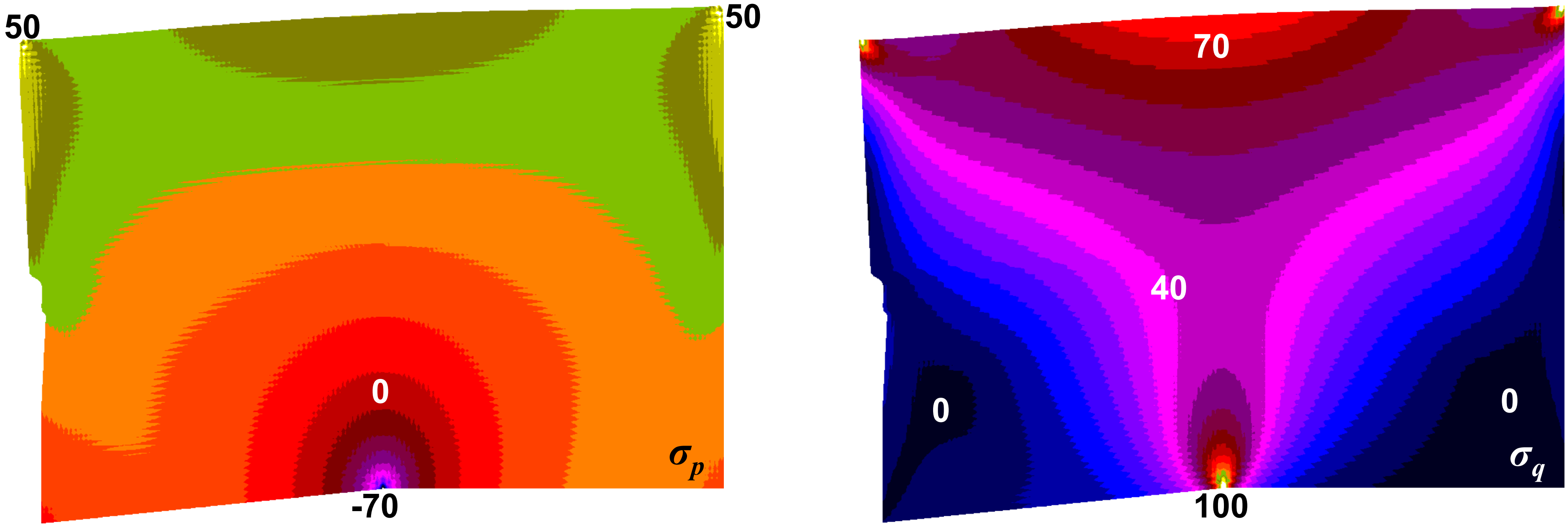
5. Examples and Discussion

Rotating sand box simulation (Visage)

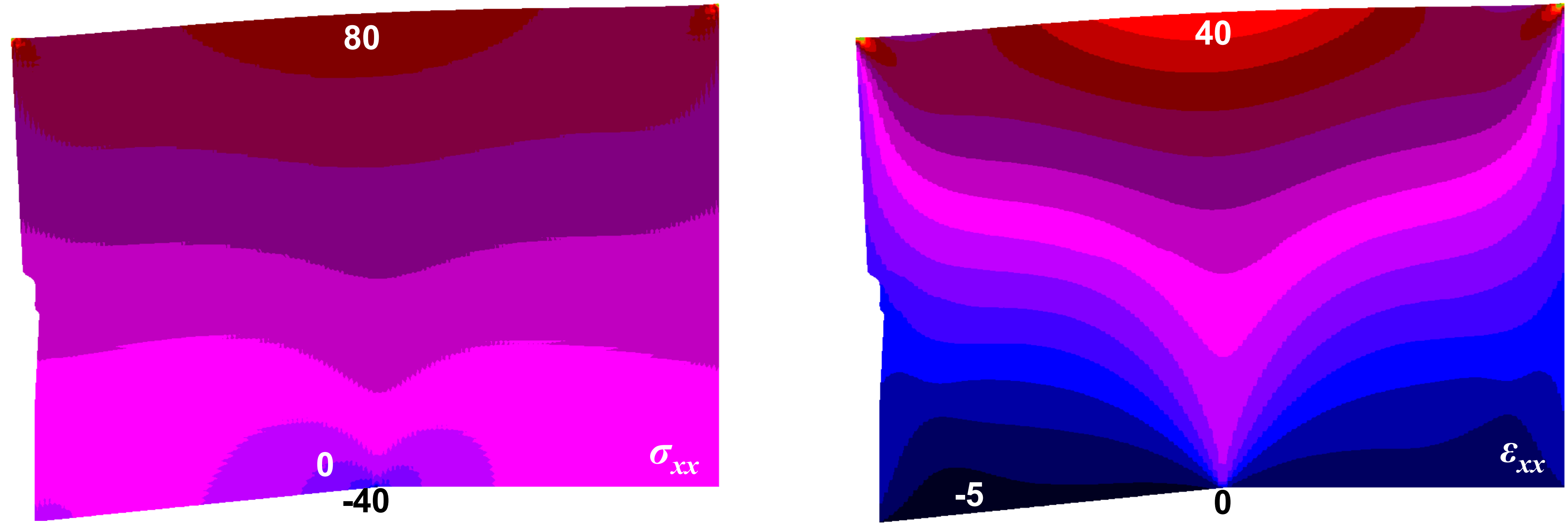


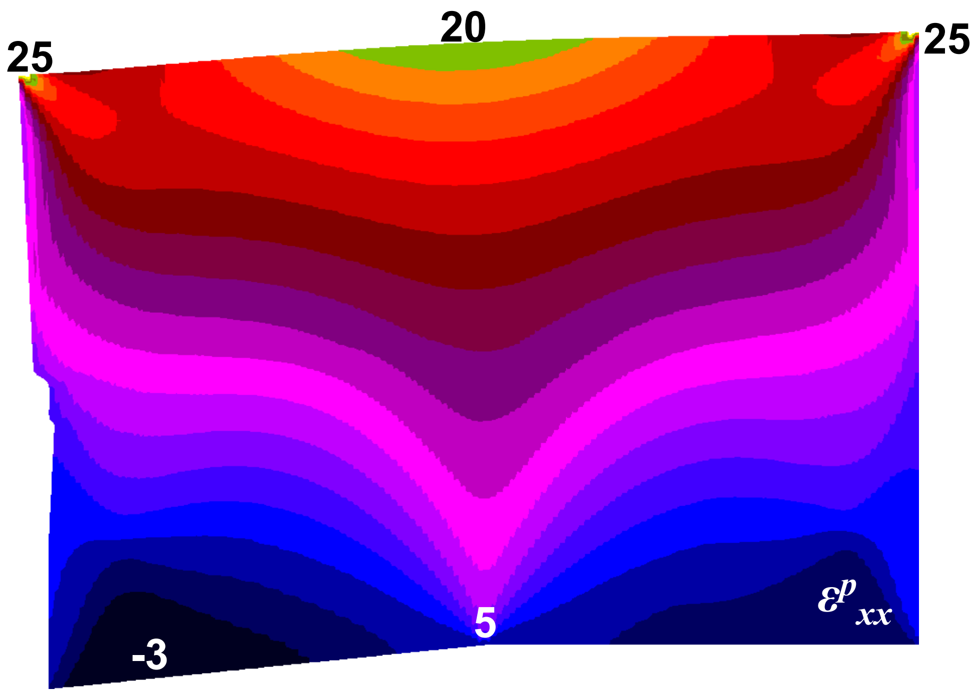


Effective stress (MPa)

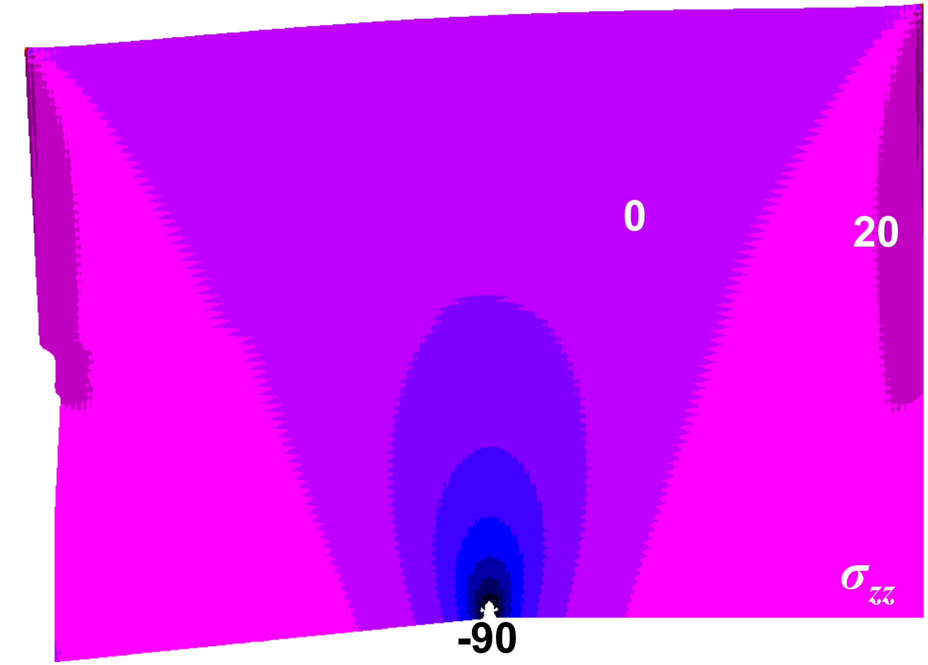


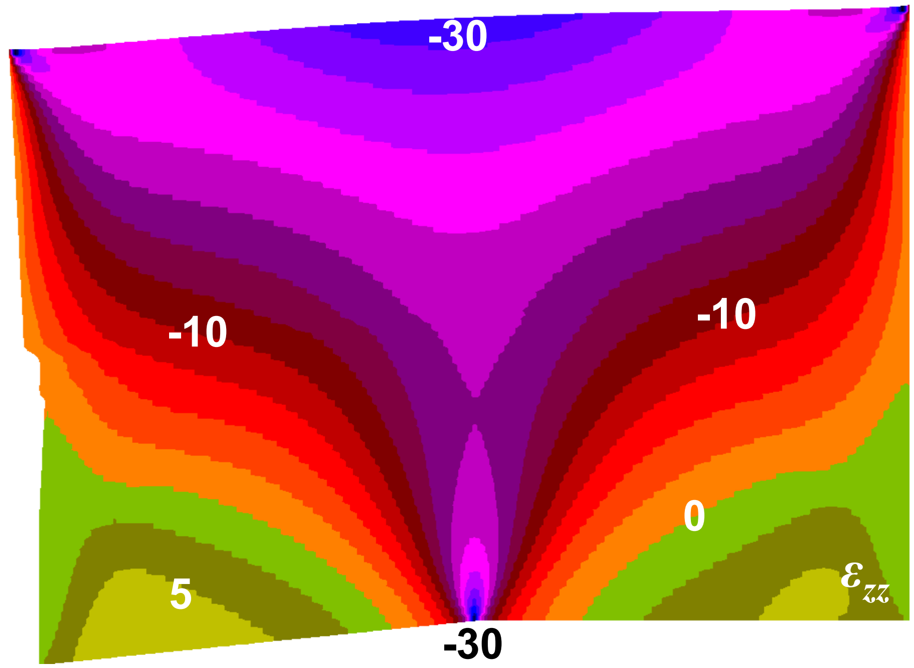
Mean & Deviatoric Effective stress (MPa)

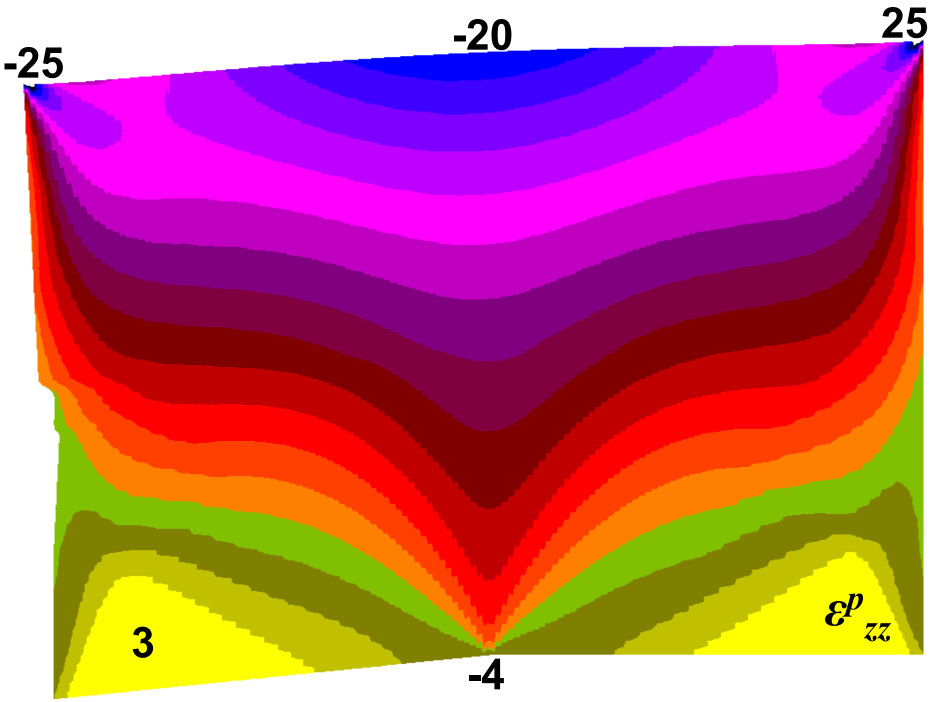




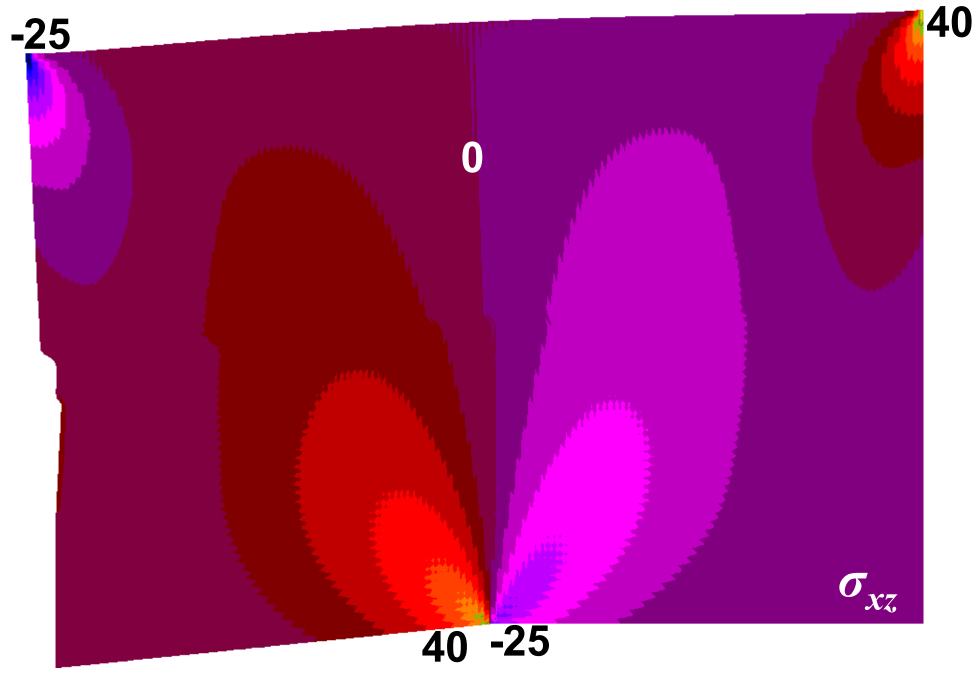
Effective stress (MPa) vs.
Total and plastic strain, (millistrain)
xx-component

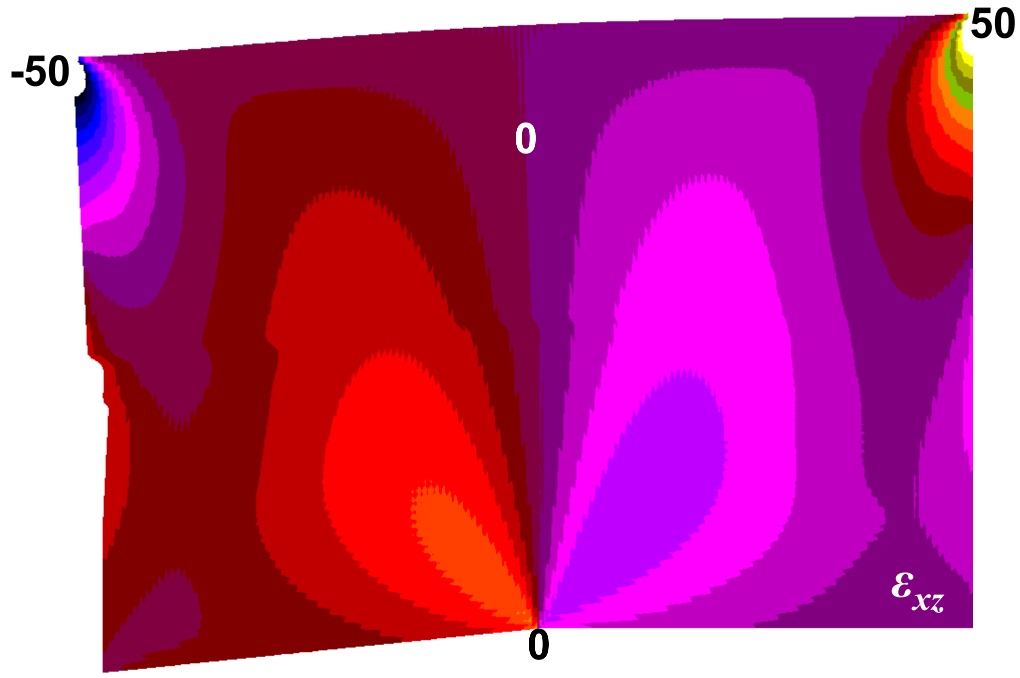


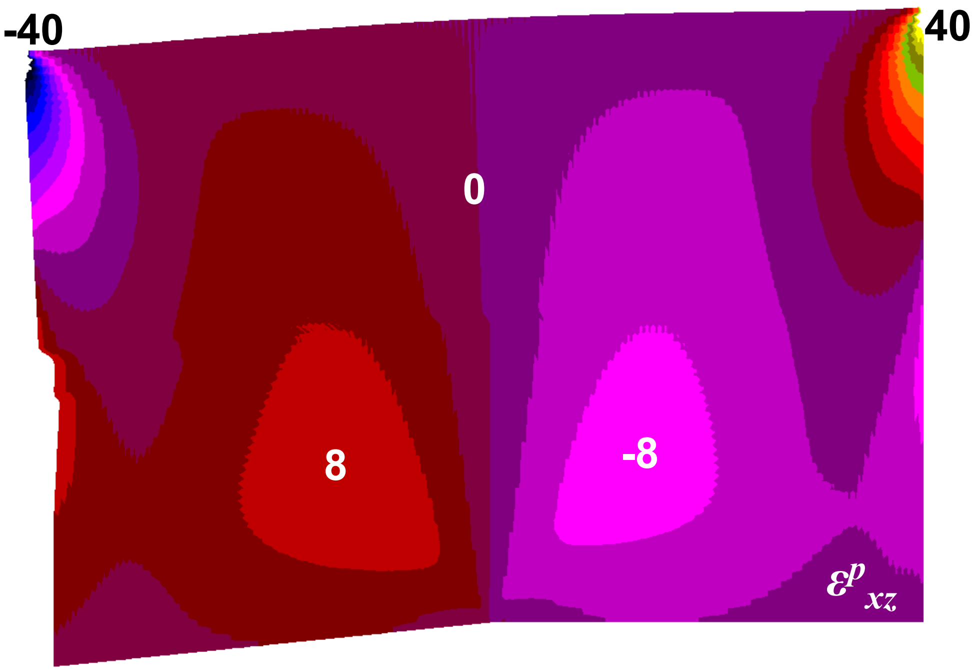




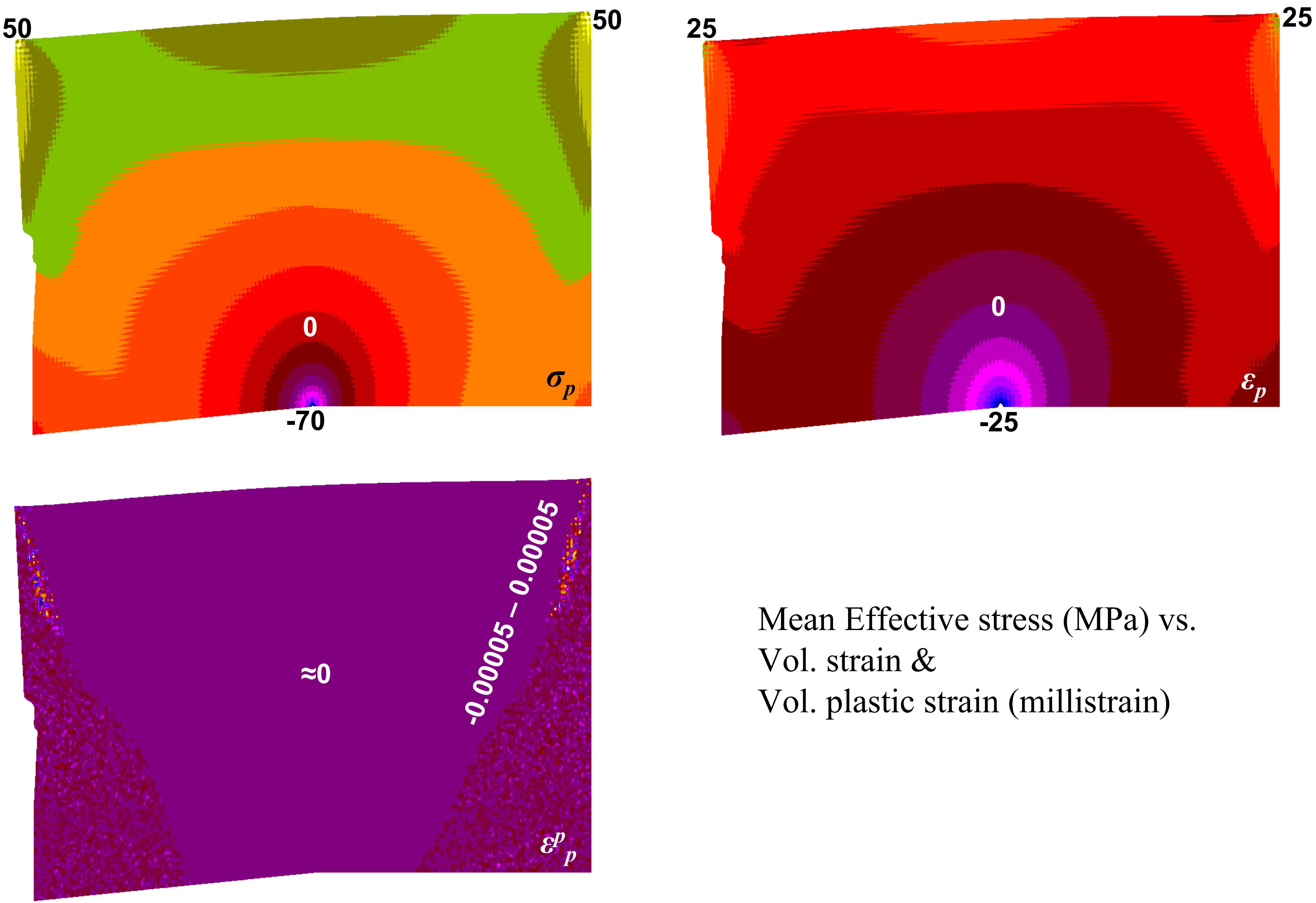
Effective stress (MPa) vs.
Total and plastic strain, (millistrain)
zz-component



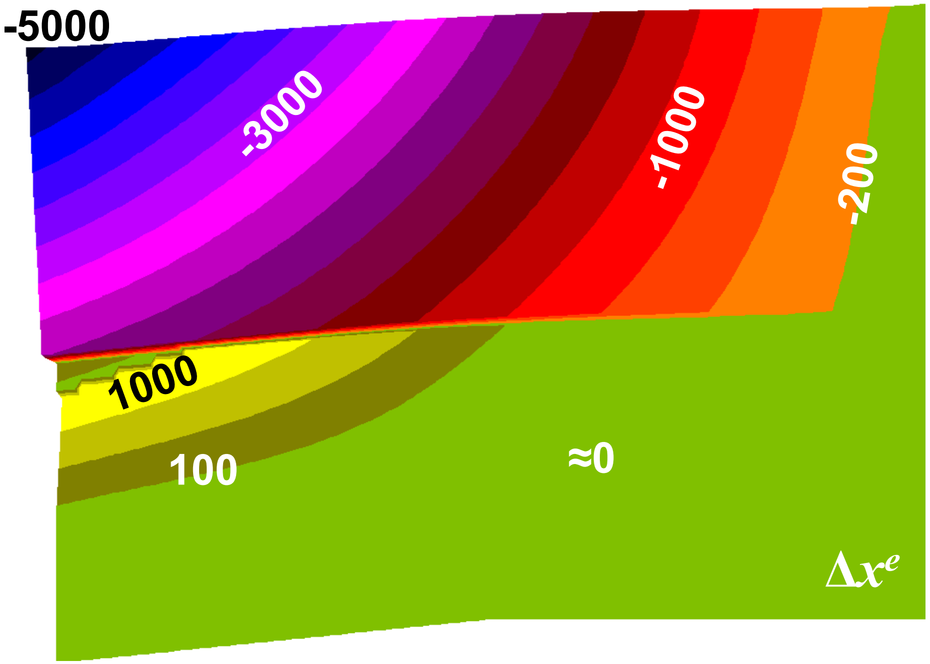


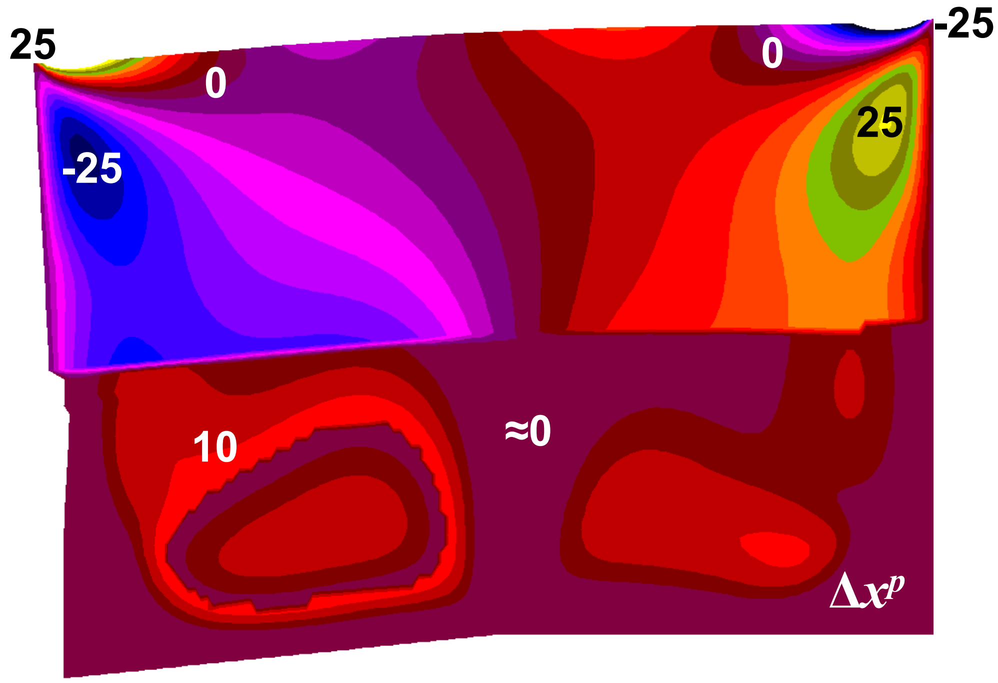


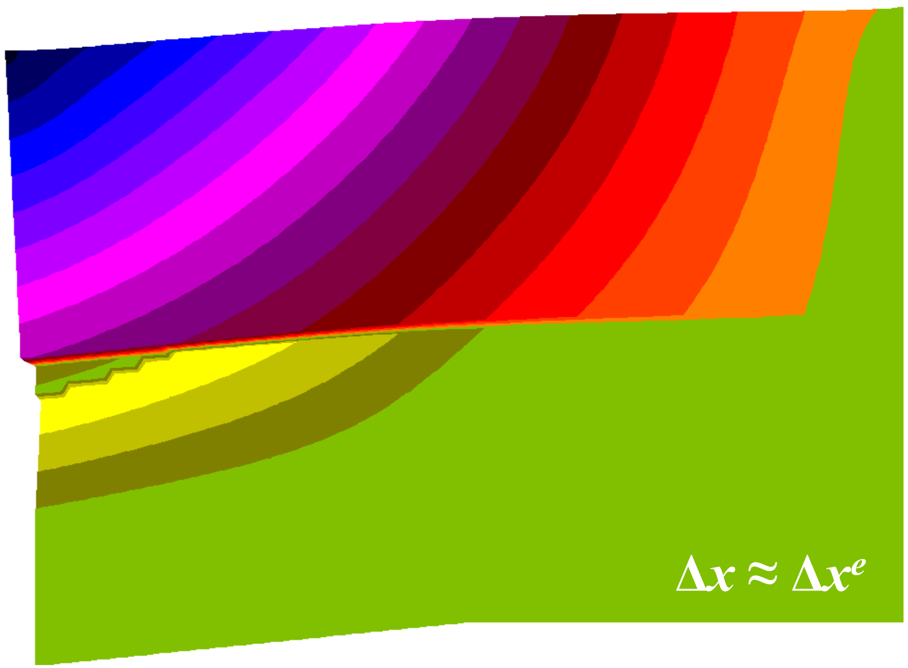
Effective stress (MPa) vs.
Total and plastic strain, (millistrain)
xz-component



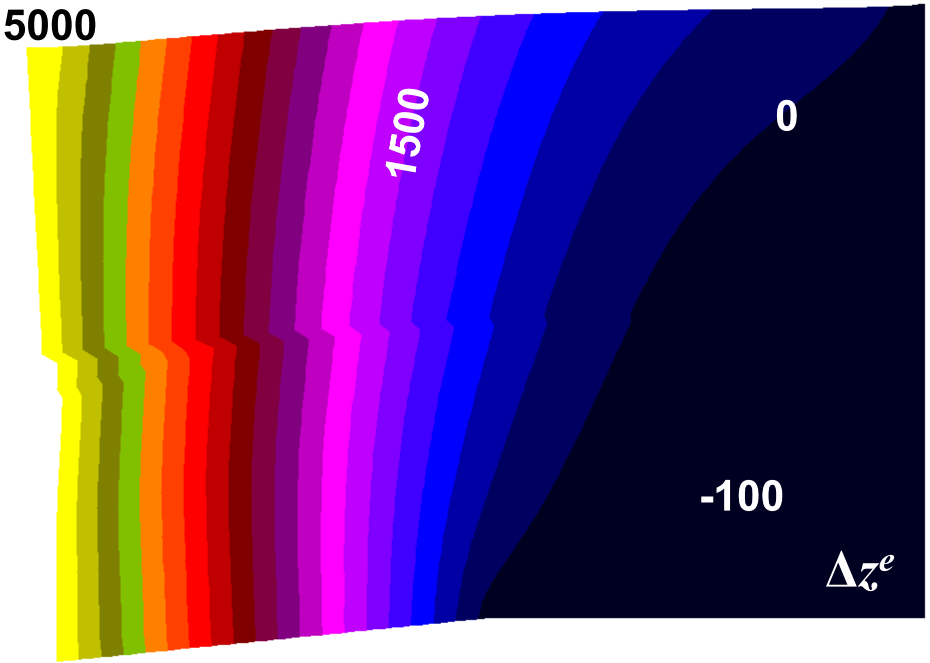
Mean Effective stress (MPa) vs.
Vol. strain &
Vol. plastic strain (millistrain)

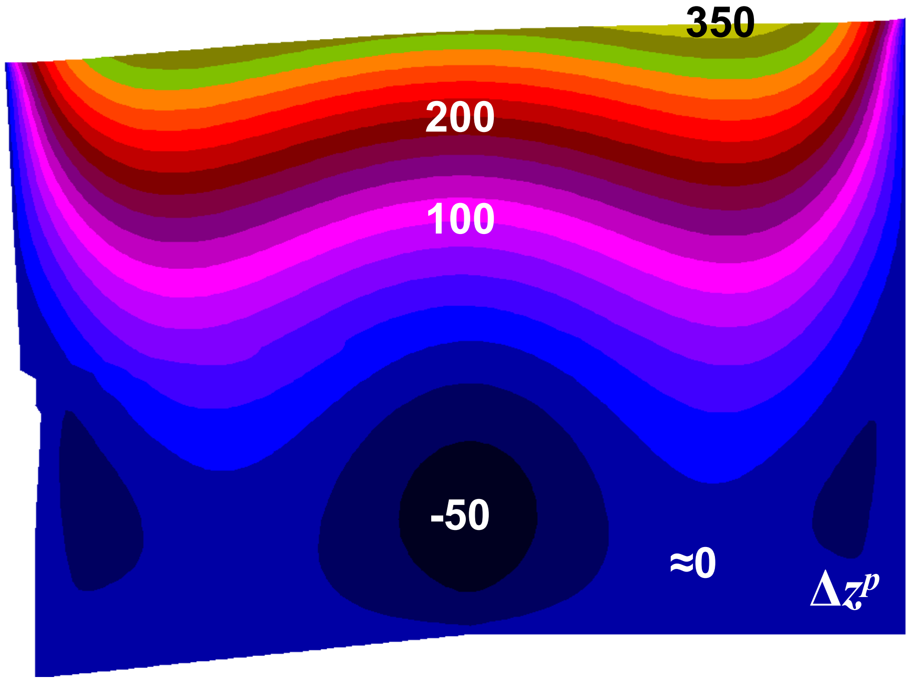


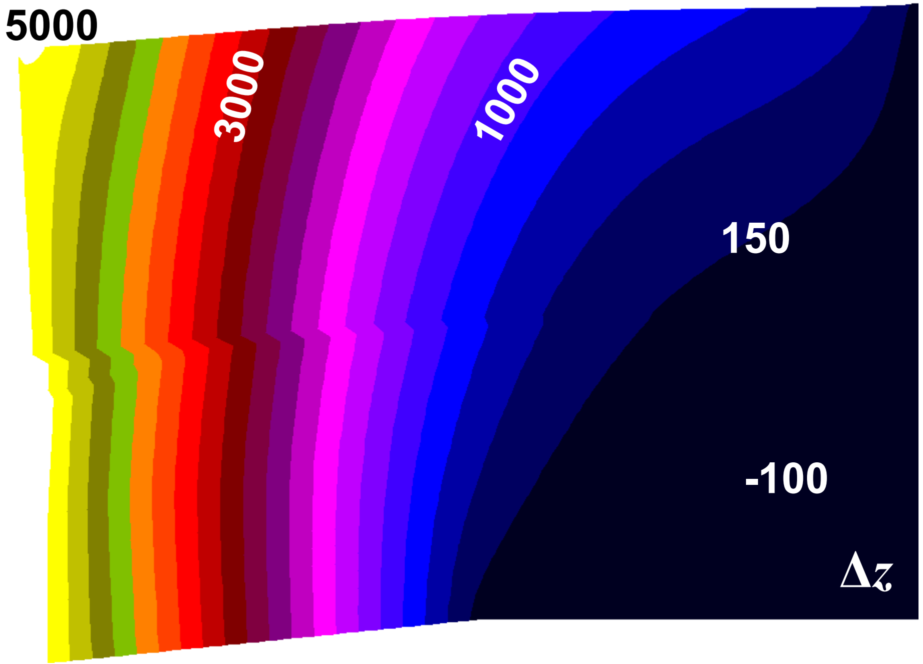




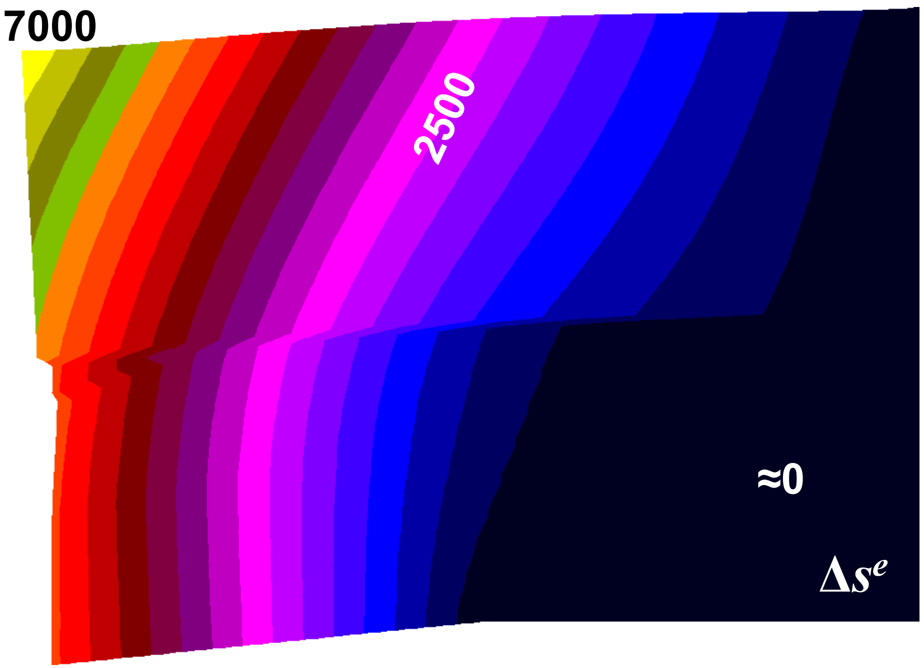
Displacements Δx (mm)
Elastic, plastic & total

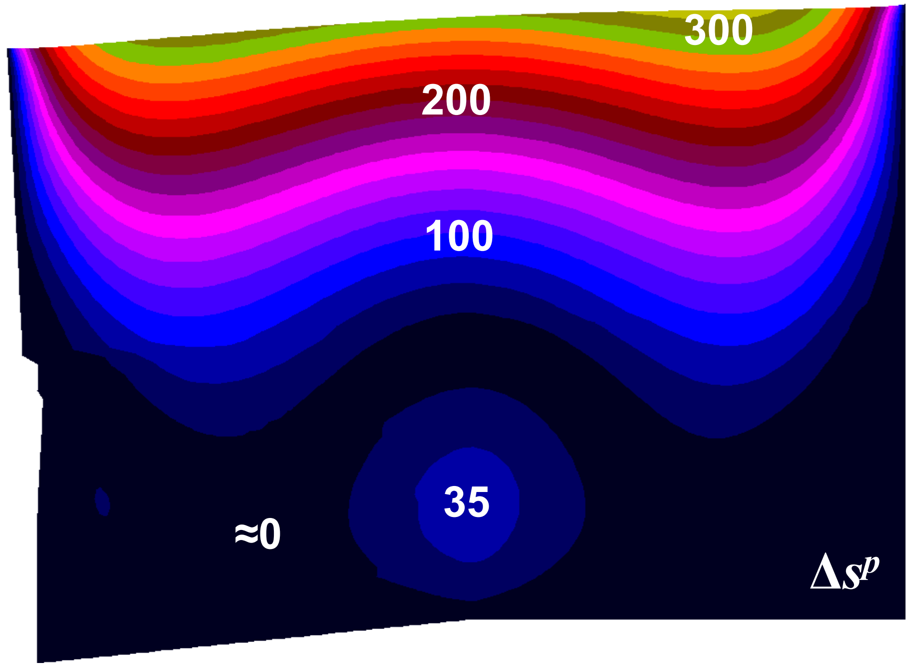


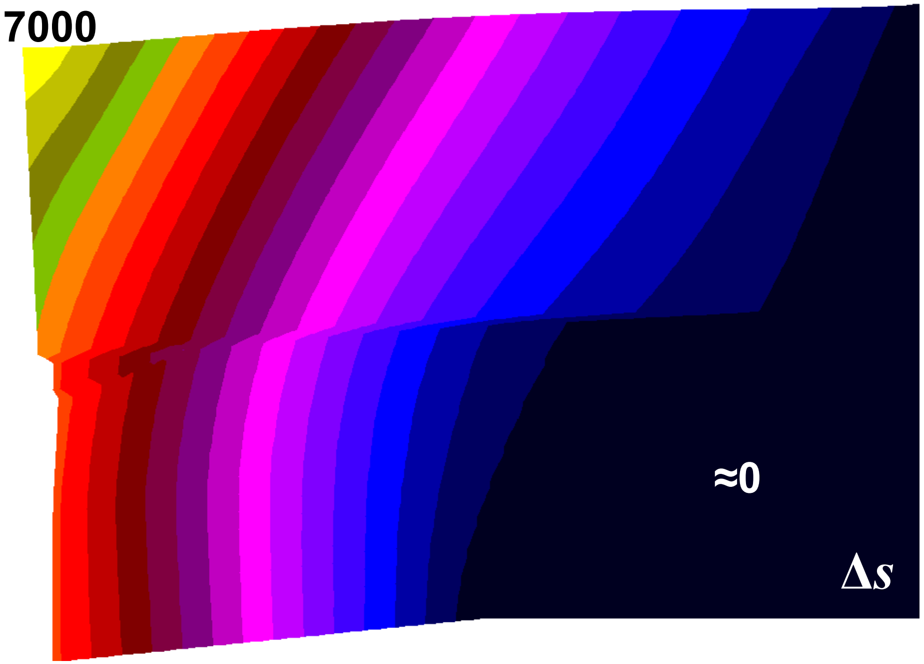




Displacements Δz (mm)
Elastic, plastic & total

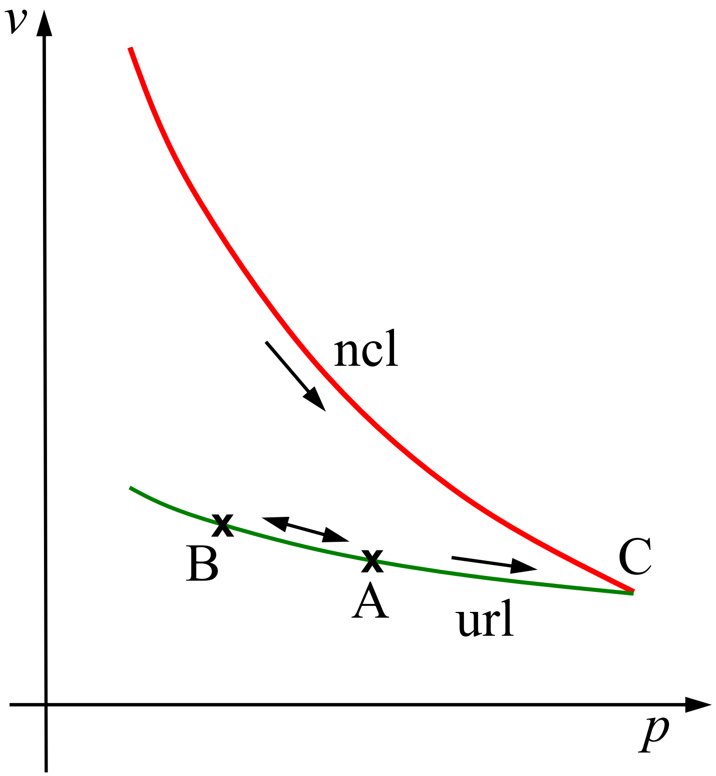


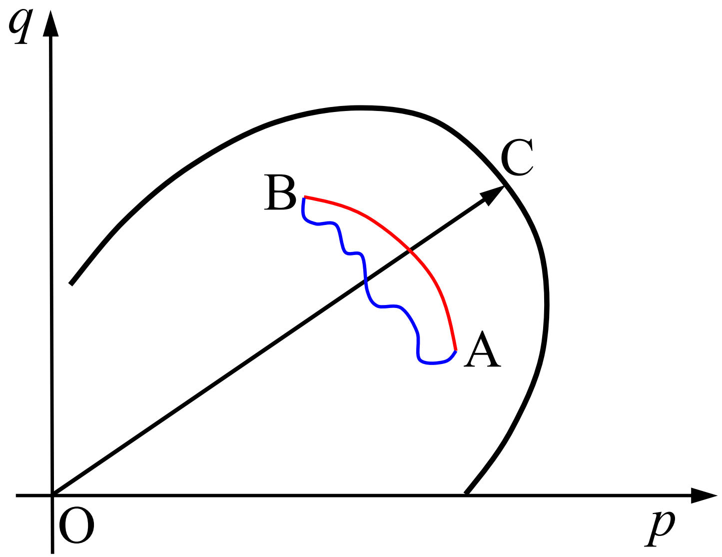




Total displacements Δs (mm)
Elastic, plastic & total

From seminar 4: Yield loci & elastic region



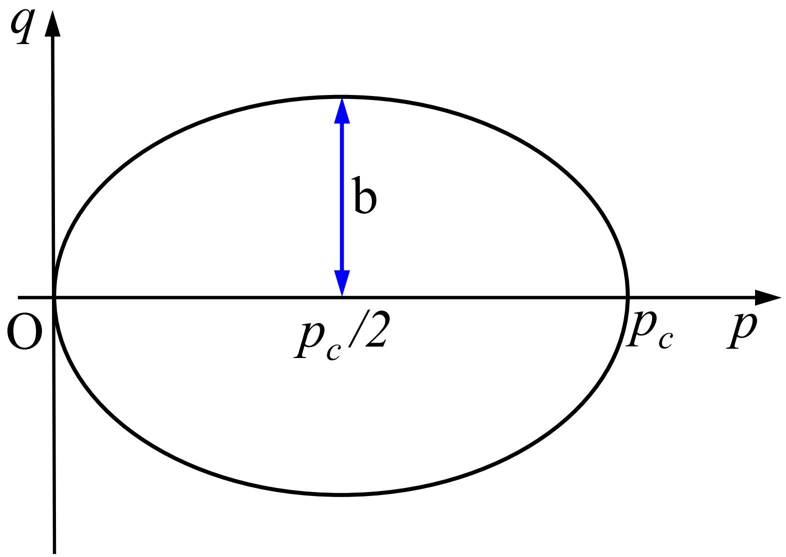


NCL: $v = v_{\lambda} - \lambda \ln p$

URL: $v = v_{\kappa} - \kappa \ln p$

Cam Clay Model

We assume a particularly simple form of the yield loci:
Ellipses with constant eccentricity (shape conserving), passing through the origin and critical mean effective stress (prev. called p_0)
Further we assume associated flow.



Different conventions exist for defining b (they all amount to the same).
Following Wood:

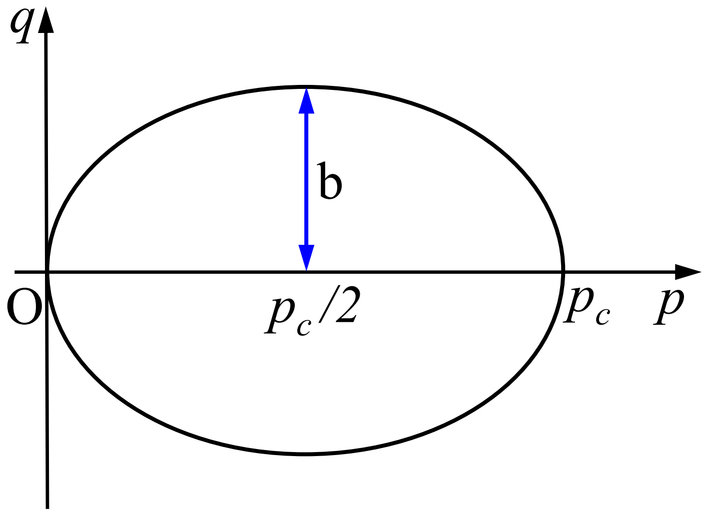
$$b = \frac{Mp_c}{2}$$

Then

$$\frac{p}{p_c} = \frac{M^2}{M^2 + \eta^2} \quad \text{where } \eta = \frac{q}{p}$$

describes a family of ellipses as prescribed

Cam Clay Model



In the general formulation, we described the yield loci (equal to plastic potentials now) by:

$f(p, q, p_c) = 0.$

For Cam Clay this becomes

$$g = f = q^2 - M^2[p(p_c - p)] = 0$$

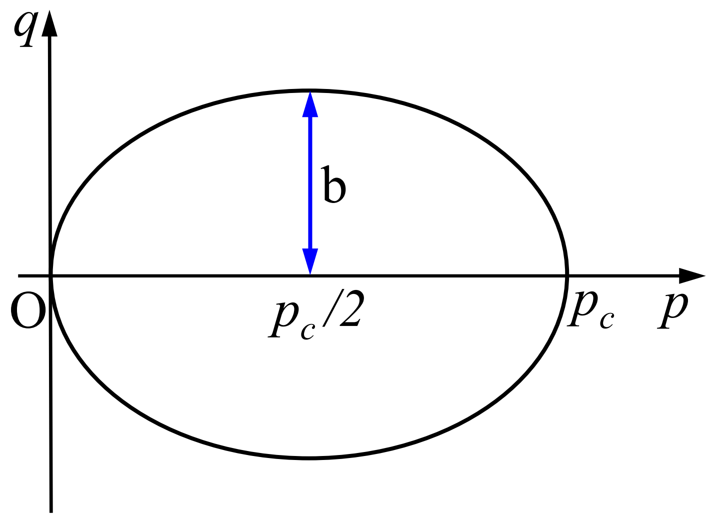
Also recall the expressions for plastic strain increments and hardening rule:

$$\delta\varepsilon_p^p = \chi \frac{\partial g}{\partial p}; \qquad \delta\varepsilon_q^p = \chi \frac{\partial g}{\partial q}$$

$$\delta p_c = \frac{\partial p_c}{\partial \varepsilon_p^p} \delta\varepsilon_p^p + \frac{\partial p_c}{\partial \varepsilon_q^p} \delta\varepsilon_q^p$$

Cam Clay Model

With our assumed form for NCL & URL:



$$\delta\varepsilon_p^p = (\lambda - \kappa)\frac{\delta p_c}{v p_c}$$

Hence

$$\frac{\partial p_c}{\partial \varepsilon_p^p} = \frac{v p_c}{\lambda - \kappa} \text{ and } \frac{\partial p_c}{\partial \varepsilon_q^p} = 0$$

Inserting these expressions into the (ugly) formula for the hardening parameter, we get

$$\chi = \frac{v_0}{\lambda - \kappa} \text{ and } \frac{\partial p_c}{\partial \varepsilon_p^p} = p\frac{v}{v_0}\chi$$

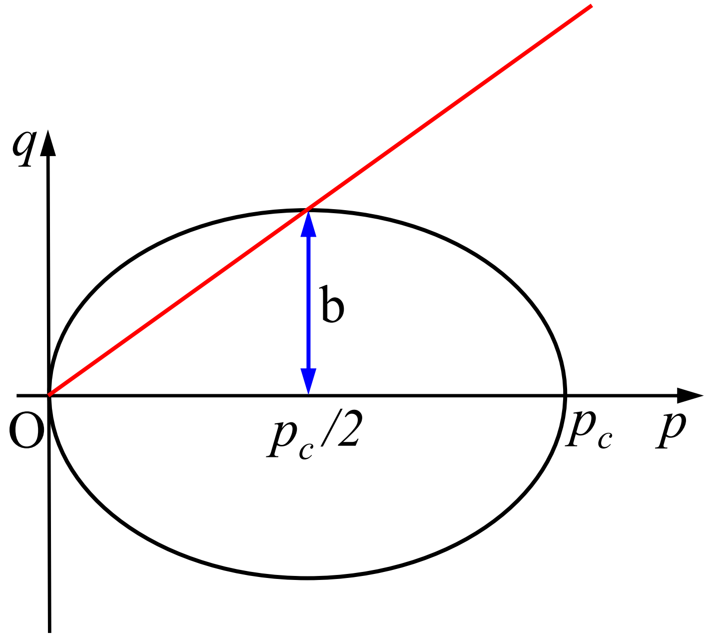
(p_{c0} = initial radius, and $v_0 = v(p_{c0})$.)

Cam Clay Model

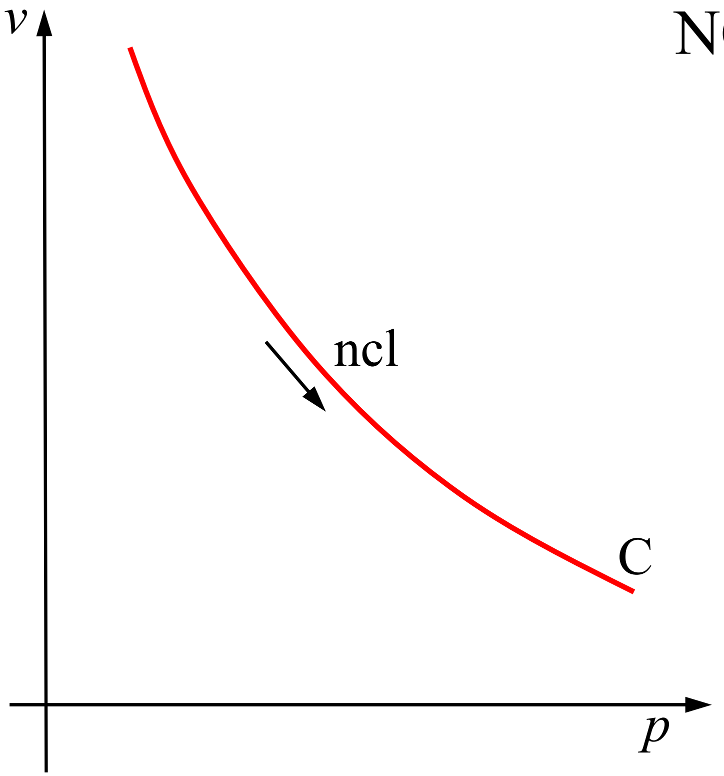
The expansion of the yield locus is then given by:

$$p_c = p_{c0} e^{\chi \varepsilon_p^p}$$

M is the slope of the red line connecting the ”top-point” of the ellipses, and can be calculated from the MC-criterion friction angle.



Parameter estimation



NCL: $v = v_\lambda - \lambda \ln p$

The bulk modulus, K (compressibility^{-1}) is a suitable parameter (often measured). When using the Cam Clay model with real data, it can be advantageous to approximate the data with a smooth curve, e.g.

$$K(p) = K_0 + a_1(p - p_0) + a_2(p - p_0)^2$$

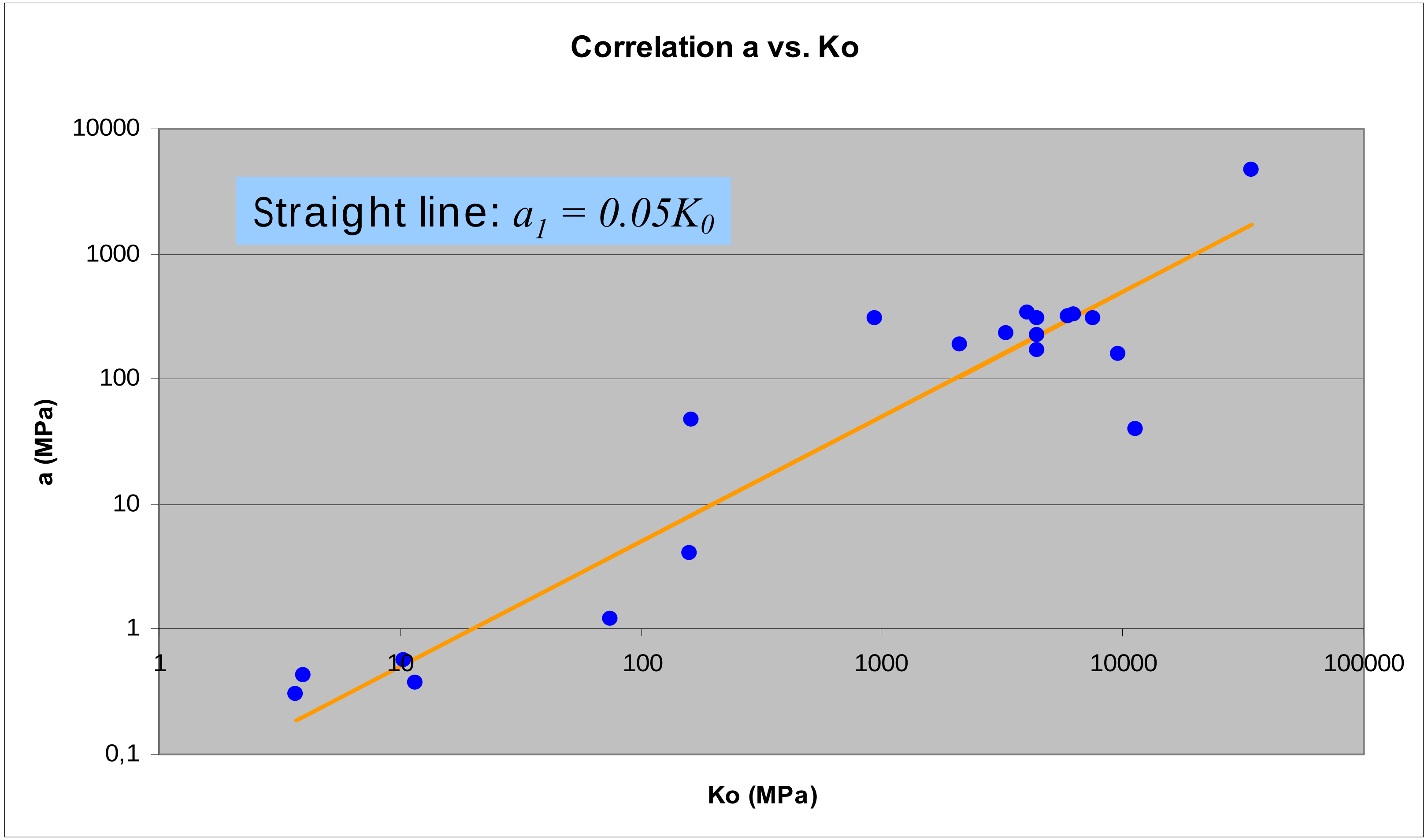
($a_1 > 0$ ensures hardening)

For real data, λ can be estimated by

$$\lambda = \frac{v(p_2) - v(p_1)}{\ln p_2 - \ln p_1}$$

where p_1 and p_2 are chosen to fit data in appropriate stress-range

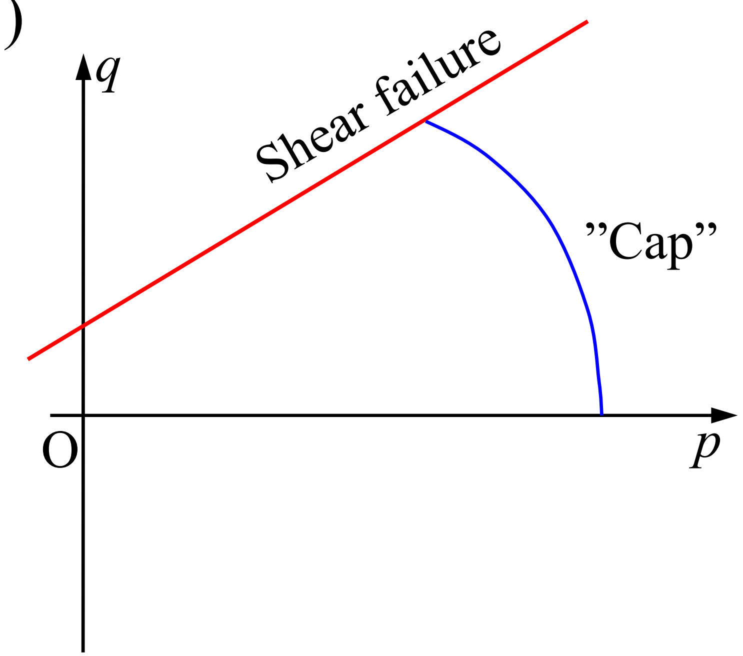
Curve fitting: a_1 depends on K_0?



”Proves” hardening of real materials under compression

Variants of Drucker-Prager

A popular model is a combination (approx) of Cam Clay and Mohr-Coulomb, often called MC with a cap. (Modified or extended D-P.)



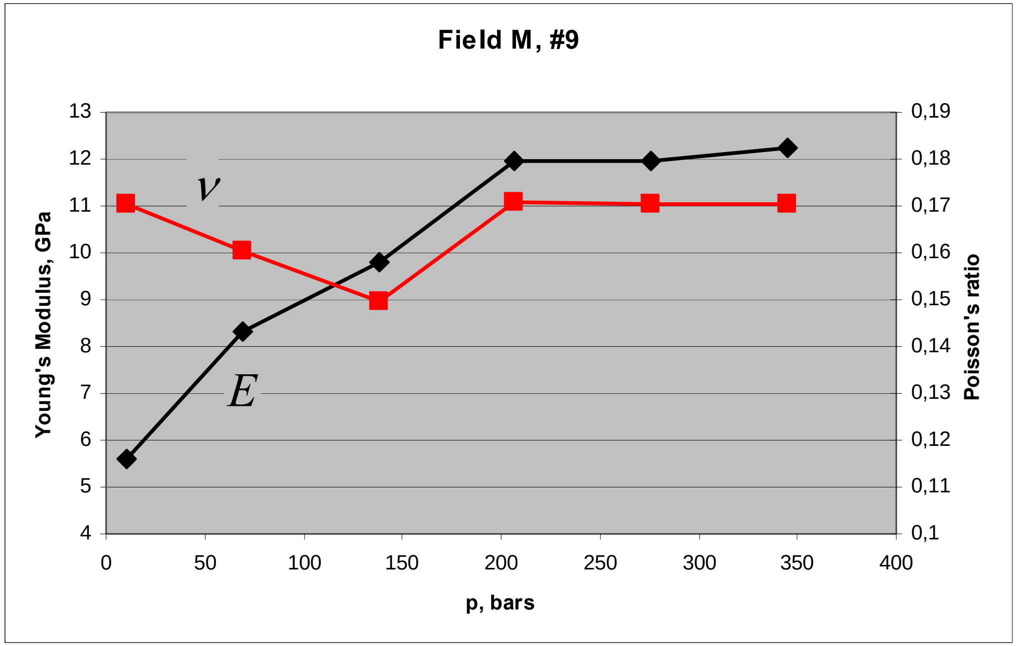
Shear failure is then described qualitatively as in MC, while expansion of the yield loci is qualitatively as in Cam Clay.

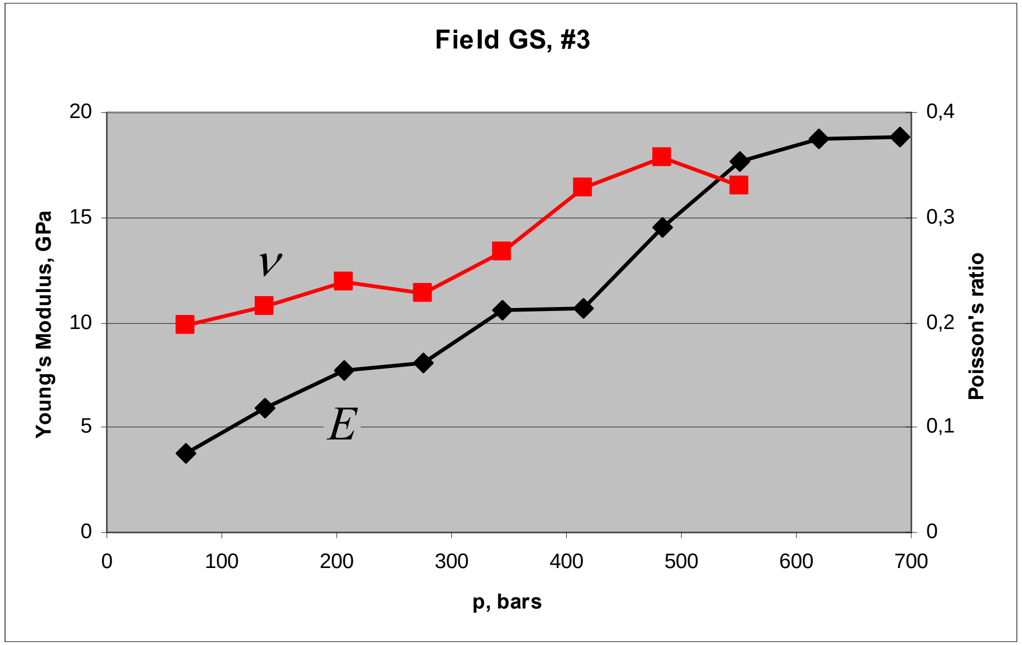
The Drucker-Prager model has many extensions / variations over this theme.

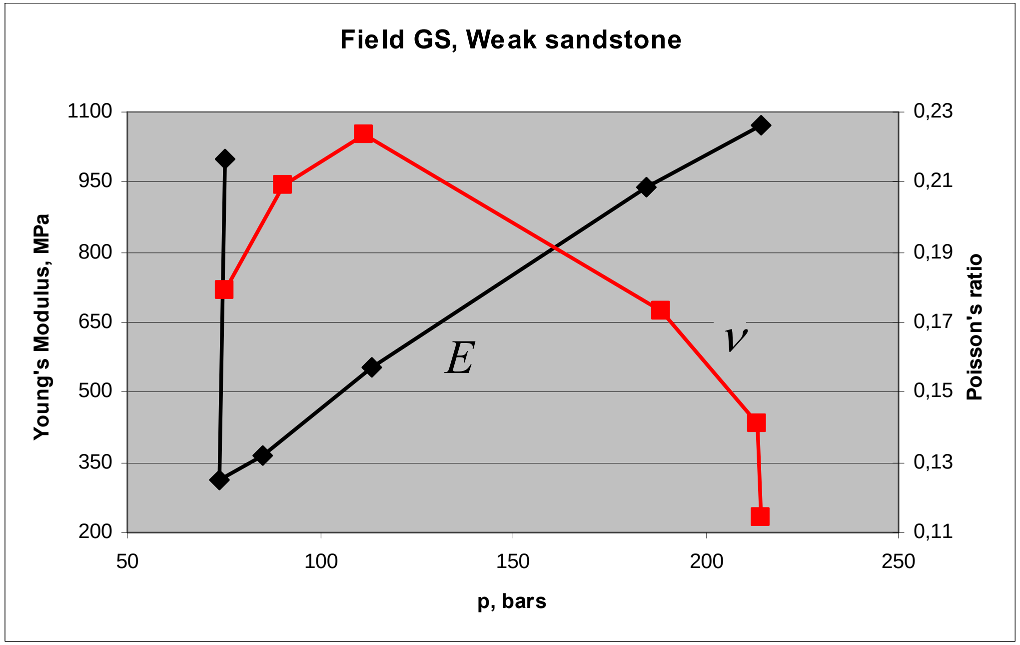
(Visage) Chalk model carries this even further, but is technically complicated without shedding new light on the theory...

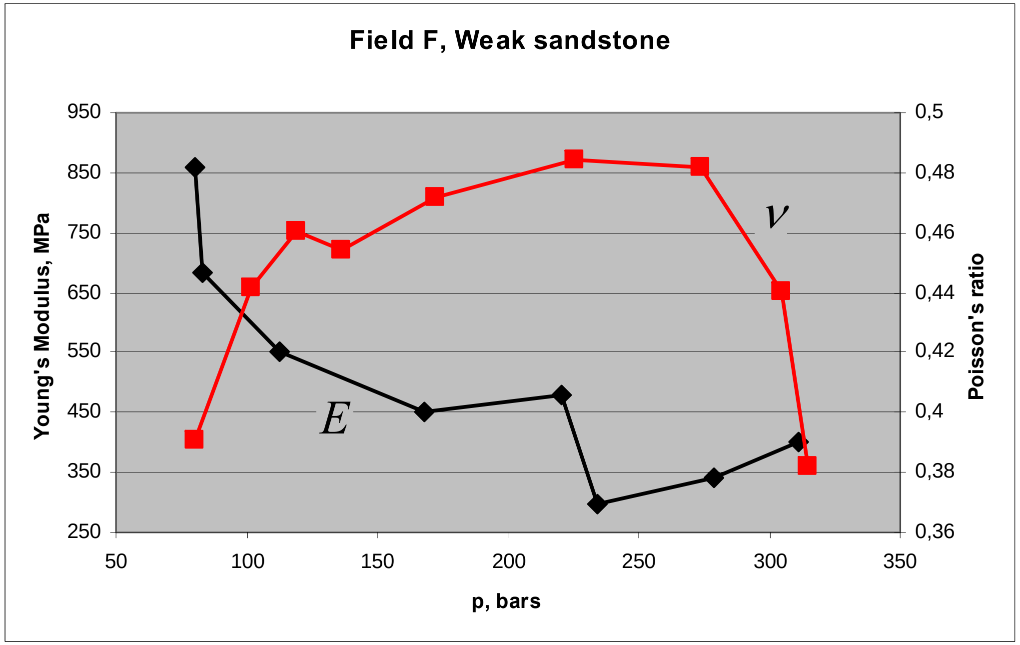
How do our idealized models compare to the real world?

Young's Modulus & Poisson's ratio – constant?

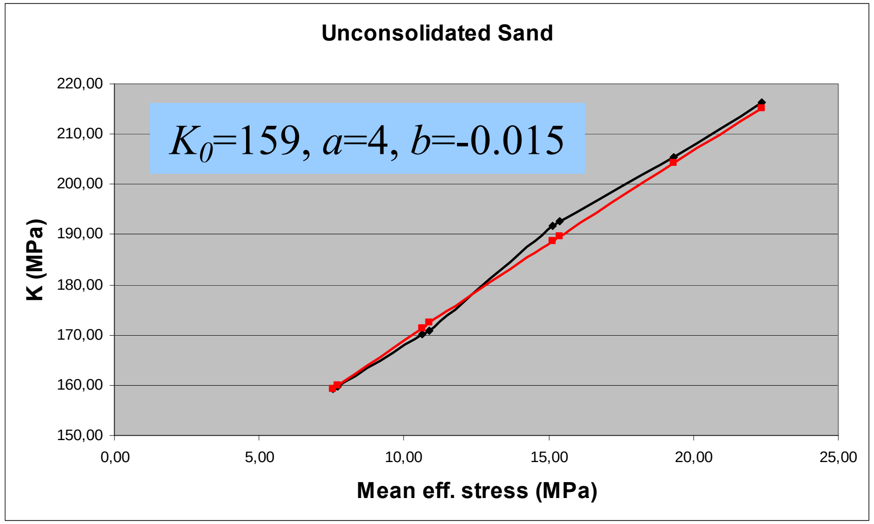


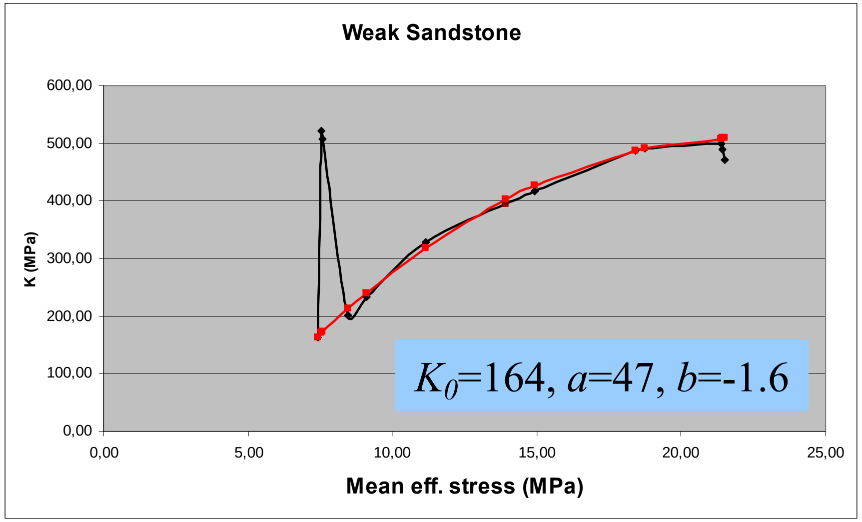


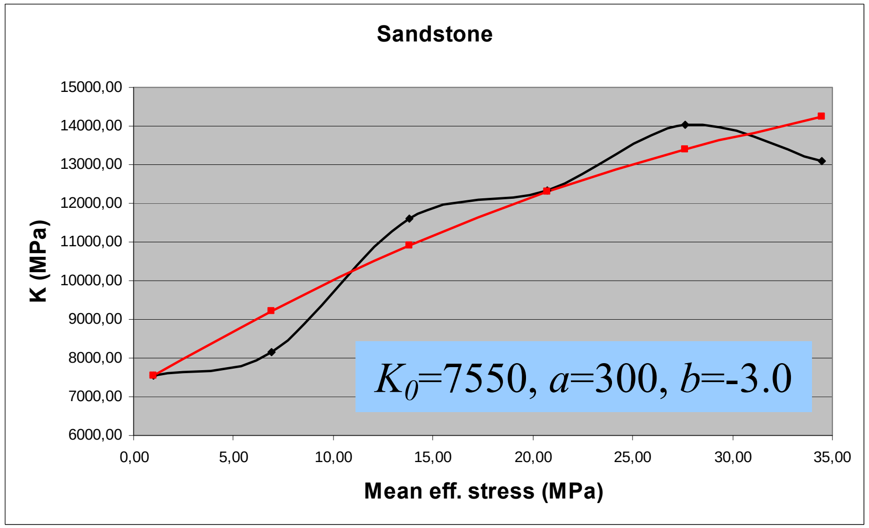


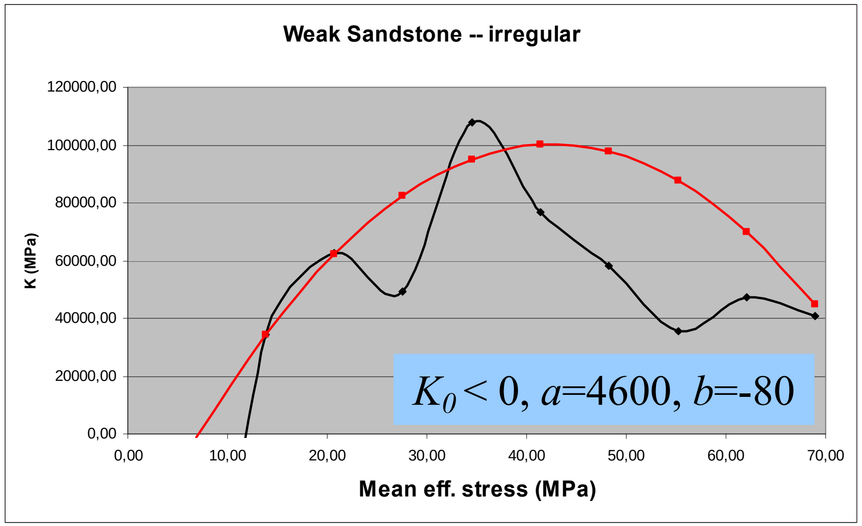


Bulk Modulus measured & polynomial approximation

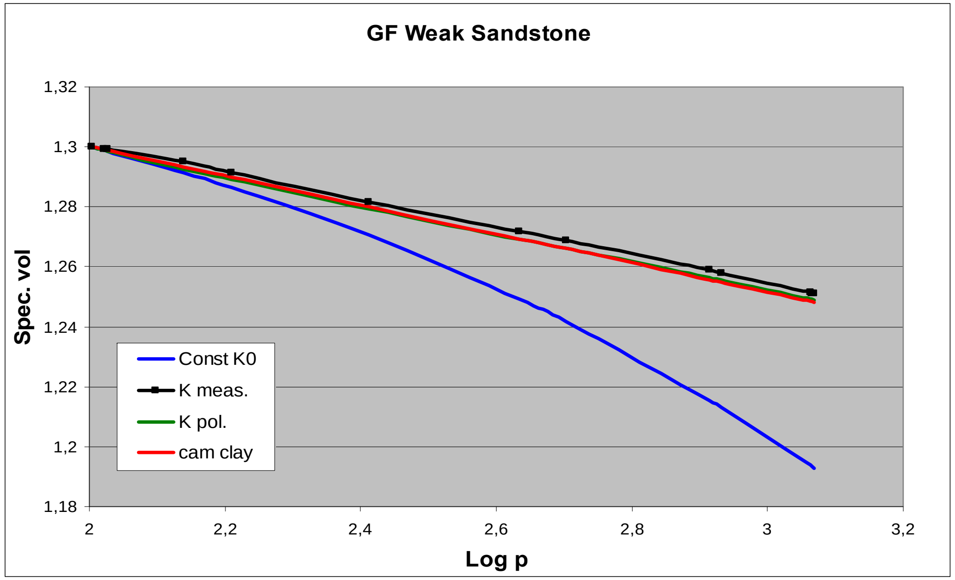


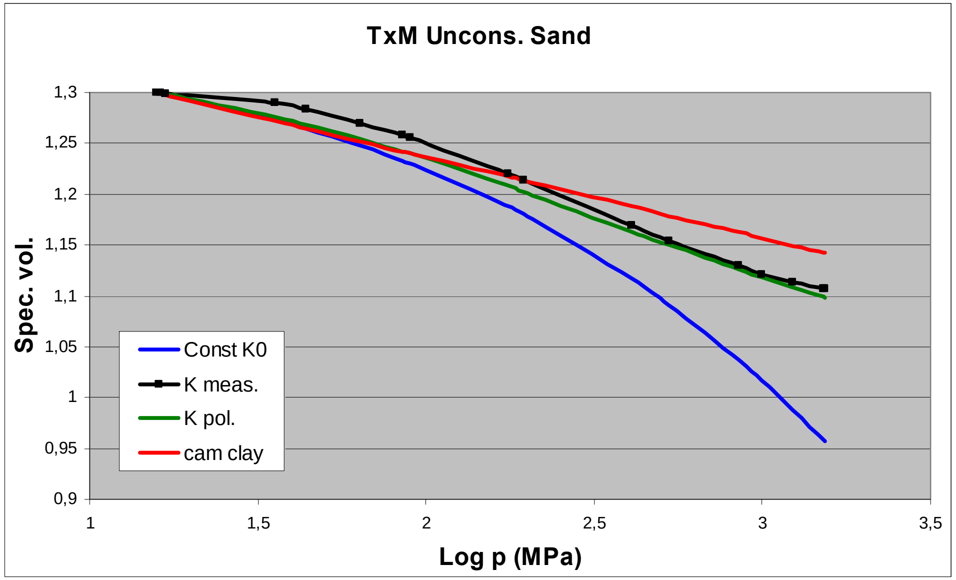


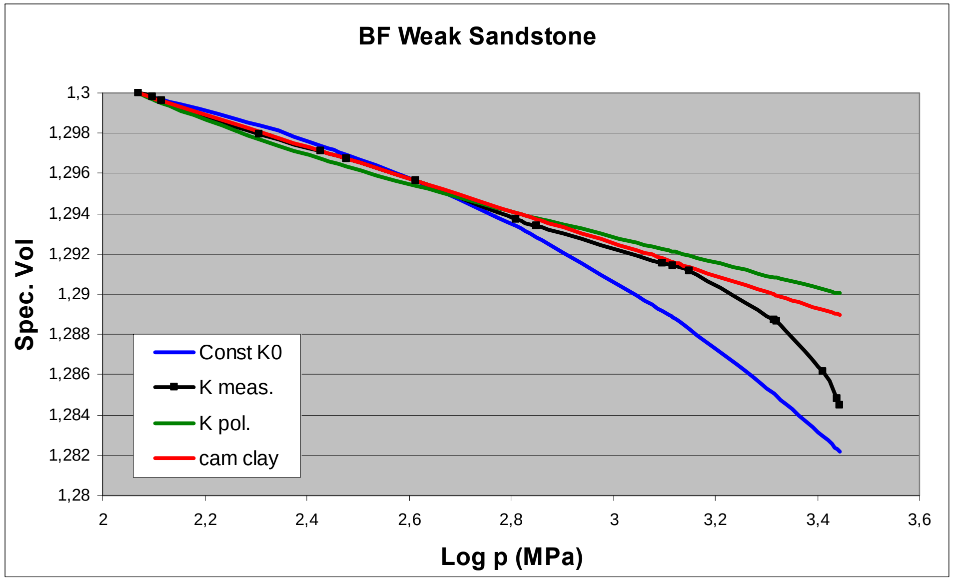


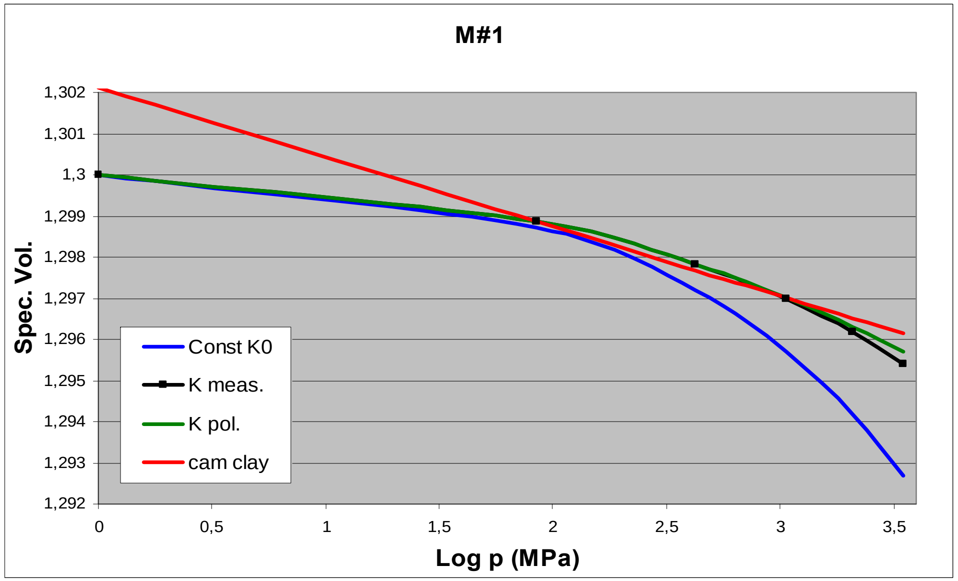


Comparison measured vs. linear elastic & Cam Clay

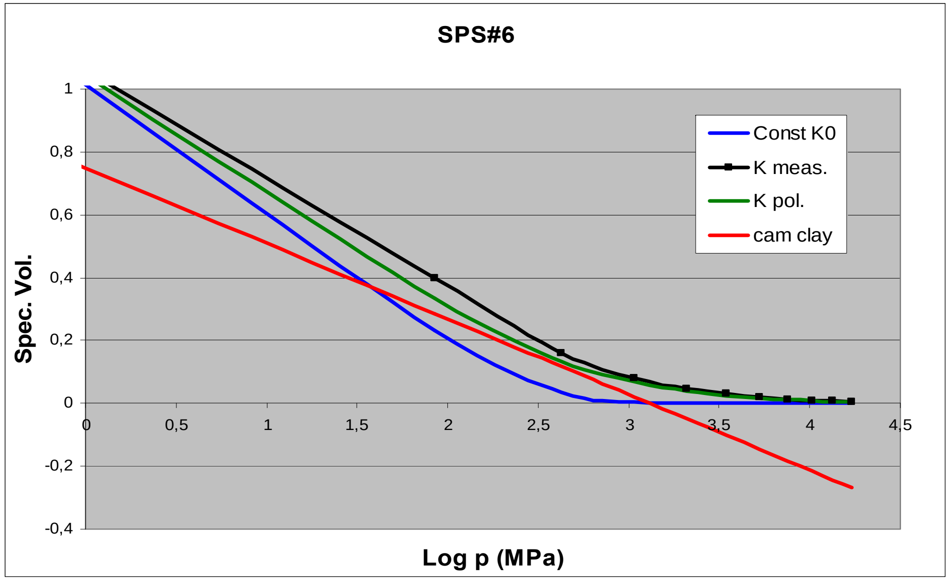


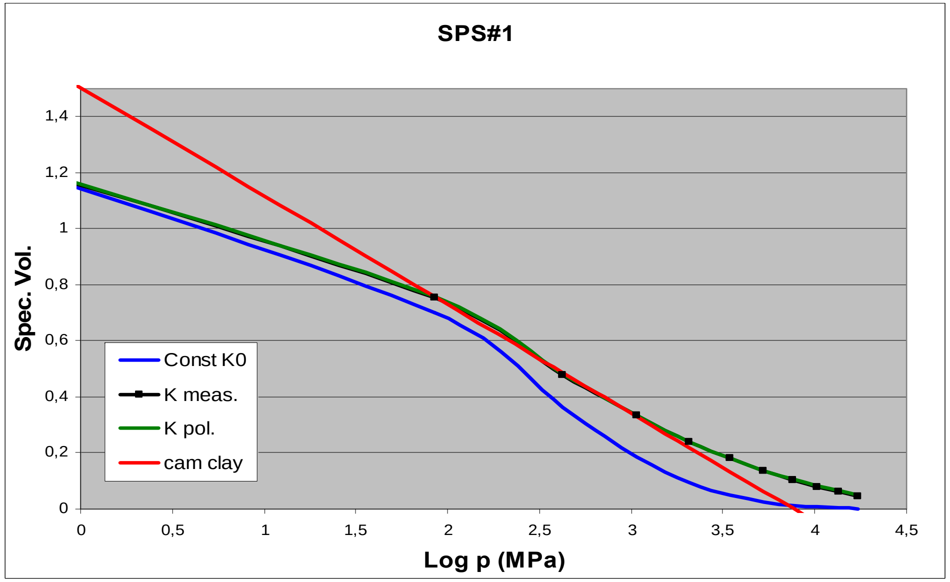


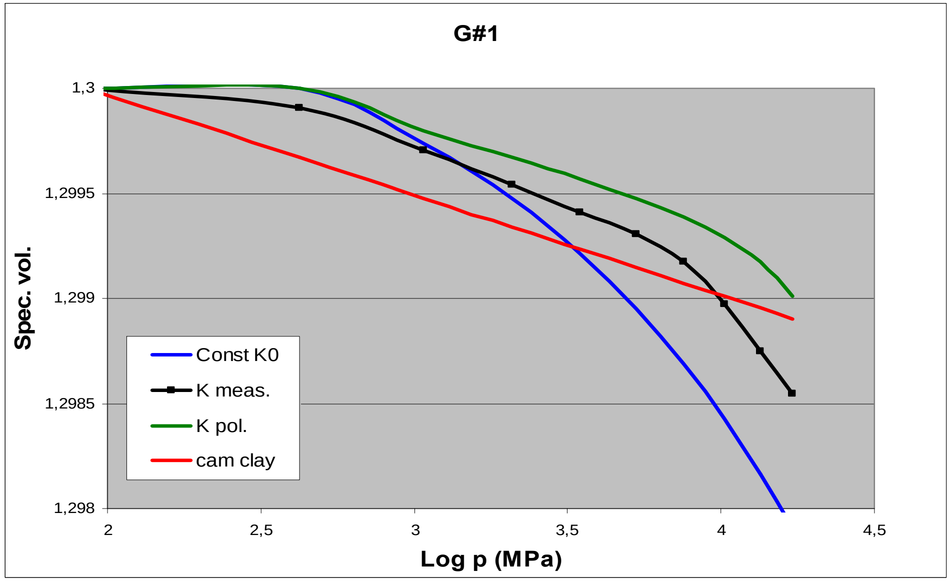




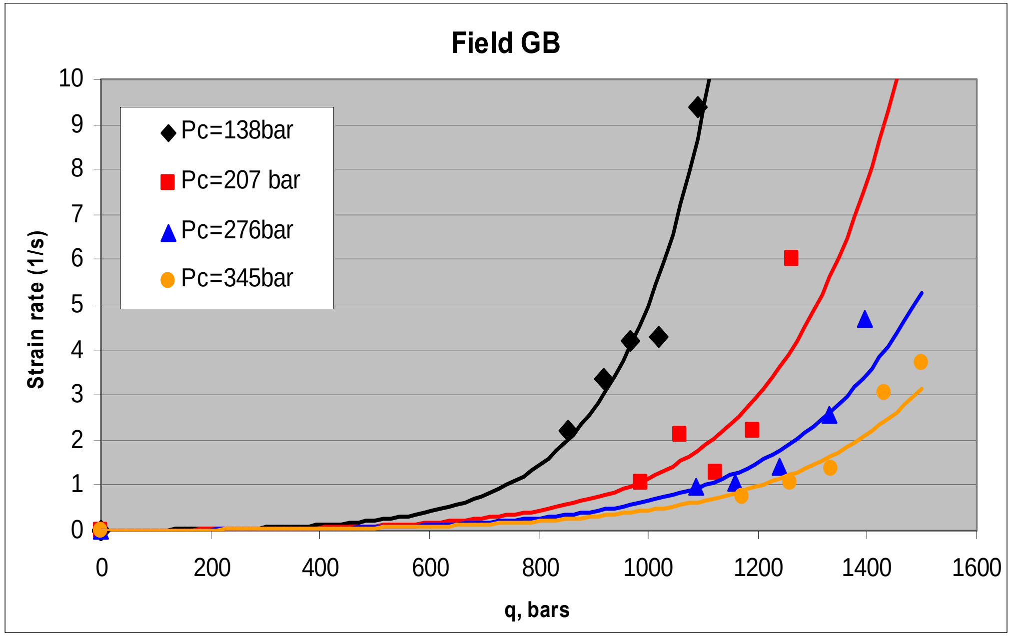
Comparison measured vs. linear elastic & Cam Clay

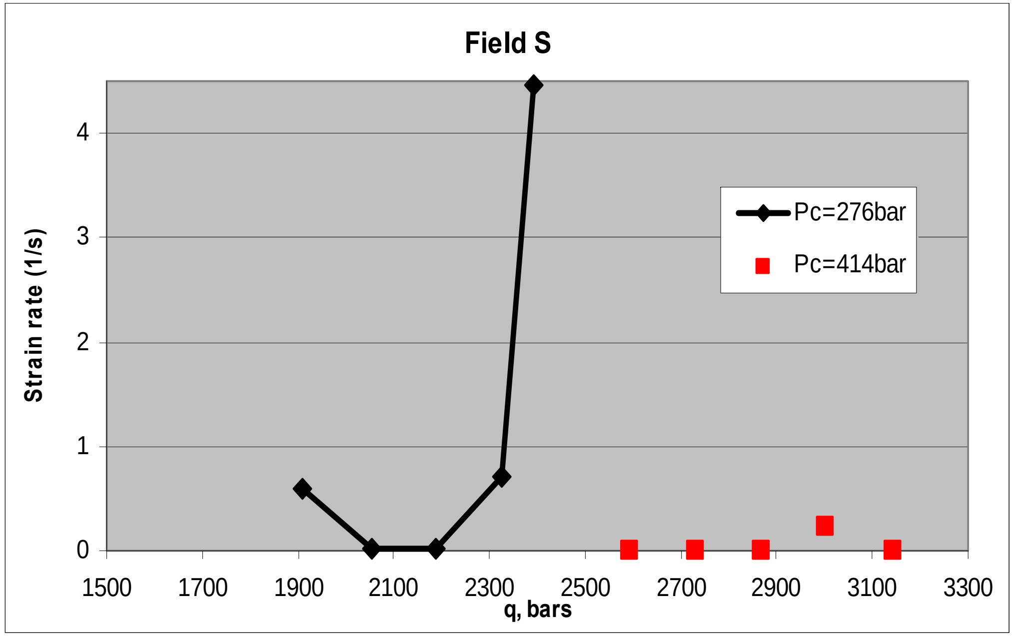


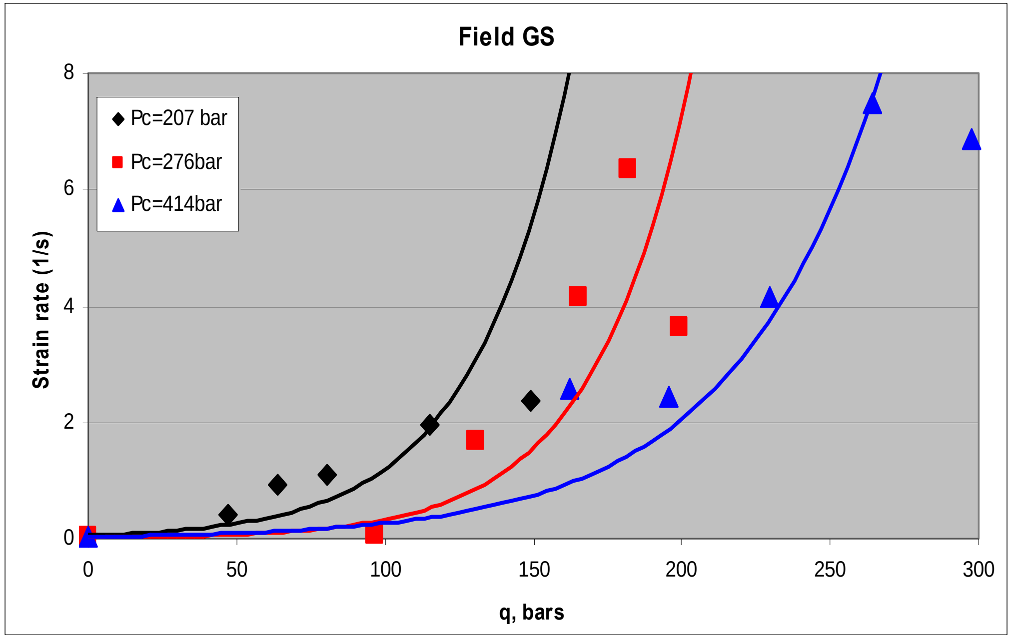




Static strain rate: Elastic or plastic behaviour?







Experiment: Apply load and leave alone, observe change in strain with time.

Recall:

Elastic: $\Delta\varepsilon$ tied to stress *increments*
(Constant $\sigma \rightarrow$ constant ε)

Plastic: $\Delta\varepsilon$ tied to stress
(ε can change when σ is constant)

GB: Elastic $\rightarrow$ plastic (normal behaviour)

S: Elastic $\rightarrow$ Failure (brittle)

GS: No elastic, immediate plastic on load