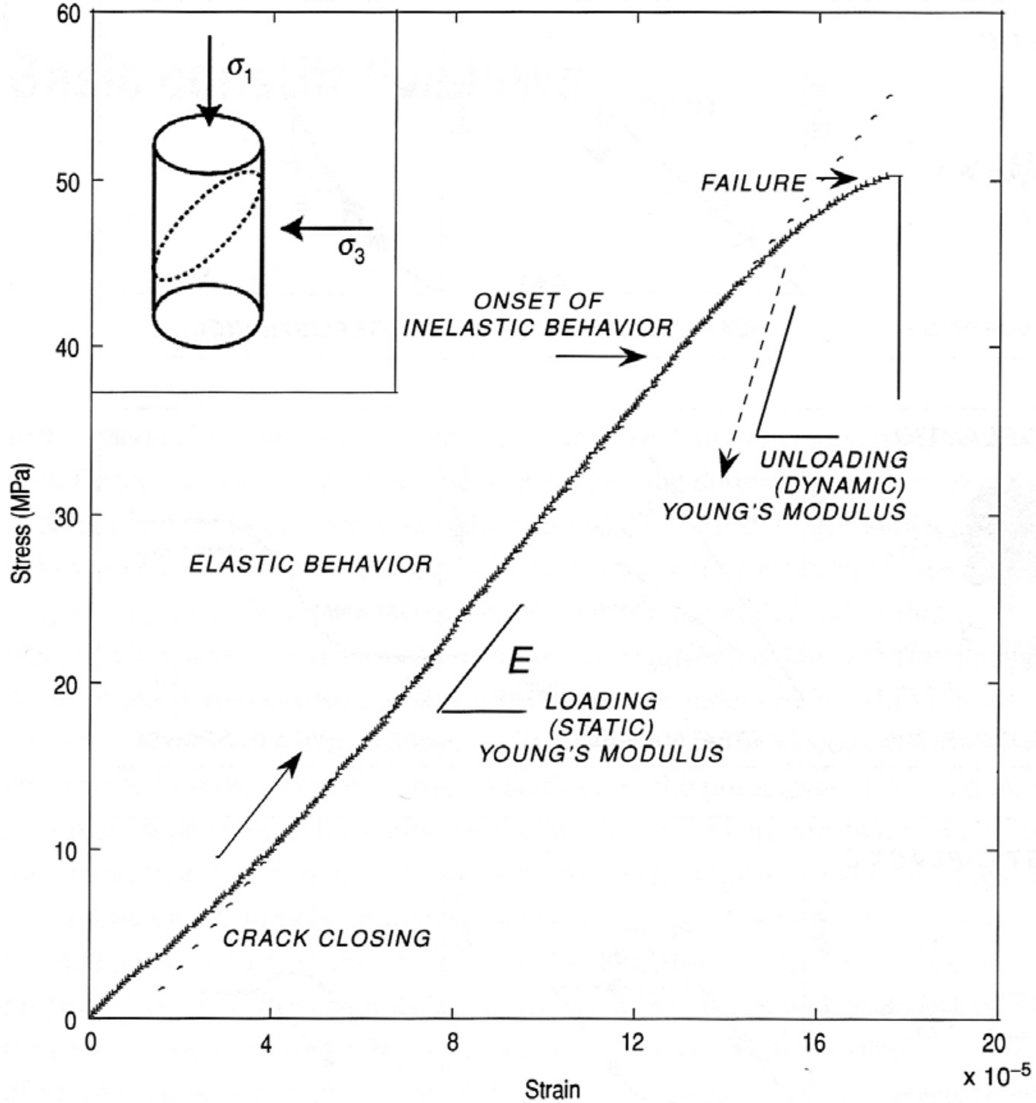
Rock Mechanics Seminar Series 2010

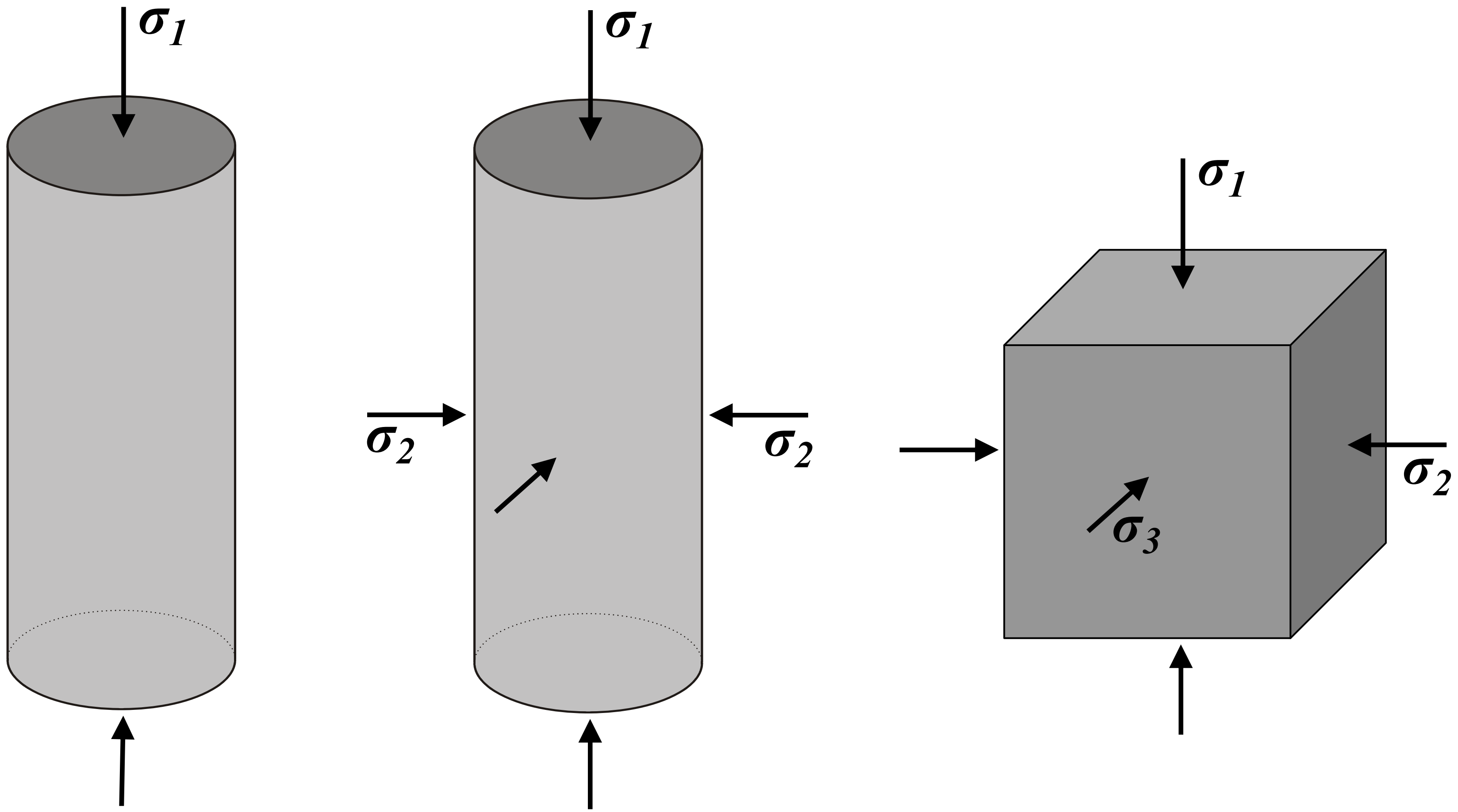
3. Failure

Load curve from prev. seminar



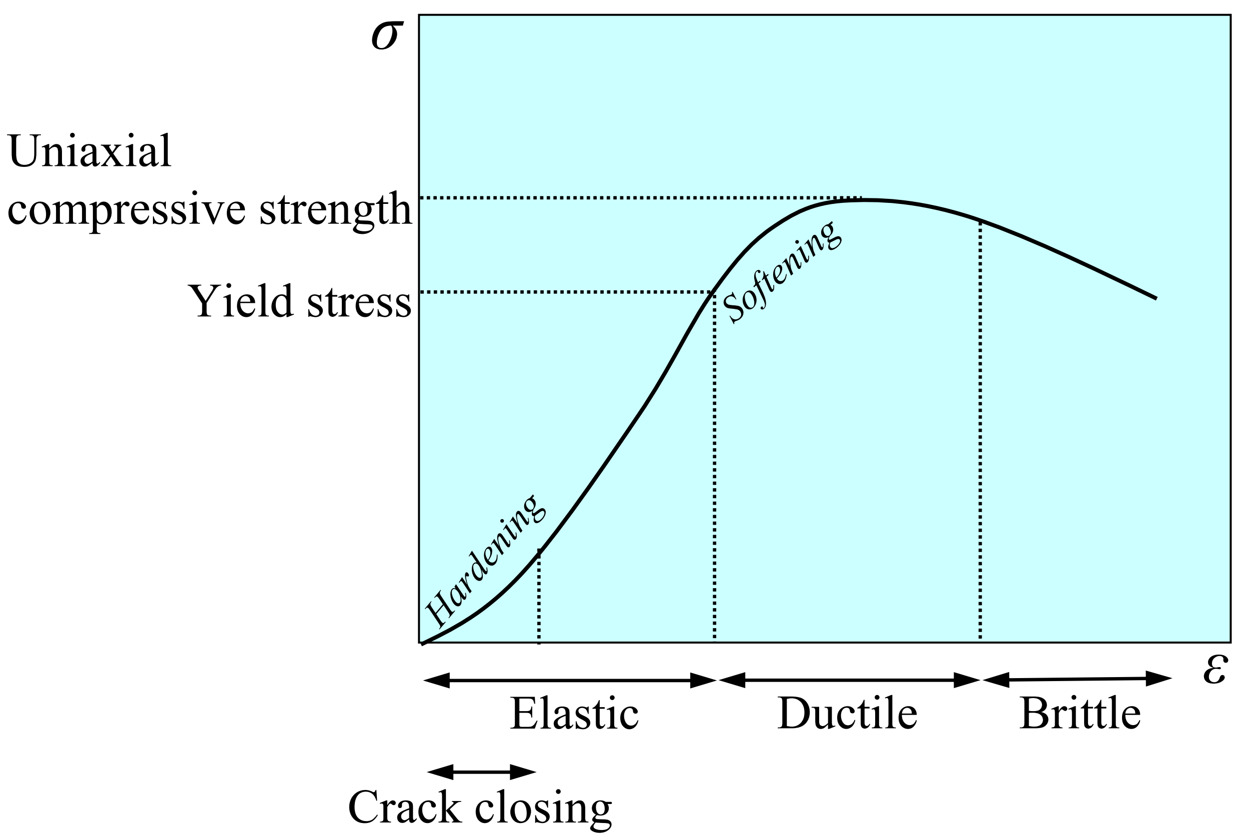
From Zoback

Common types of rock mechanics tests



Uniaxial	Triaxial	Polyaxial
$\sigma_2 = \sigma_3 = 0$	$\sigma_1 > \sigma_2 = \sigma_3$	$\sigma_1 \neq \sigma_2 \neq \sigma_3$

Ideal (theoretical) load curve

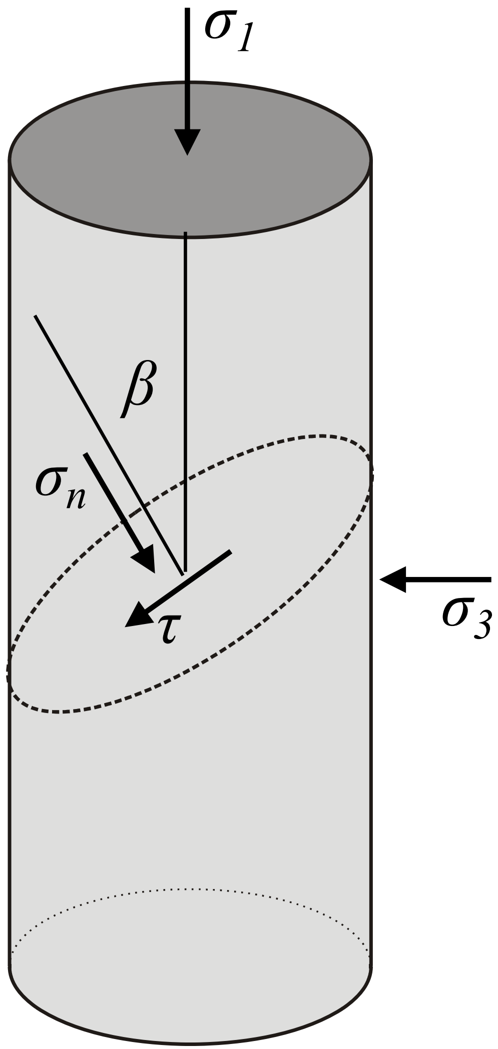


Ductile:
Permanent deformation without losing ability to support load.
Brittle:
Ability to withstand stress decreases (rapidly) with increasing deformation.
Yield point:
Beyond here permanent changes occur.
UCS:
The peak stress

Hardening / Softening;
Material's ability to support loading increases / decreases w. load.

Rightmost part of curve mostly purely theoretical, as material will fail (break) before it can be measured.

Failure of rock in compression



1. Microscopic failures
   - Creation of small tensile cracks
   - Frictional sliding
2. Coalescence of micro-failures into a through-going shear plane

Brittle: Loss occurs ”catastrophically” (Matr. loses all strength immediately.)

Ductile: More gradual failure.

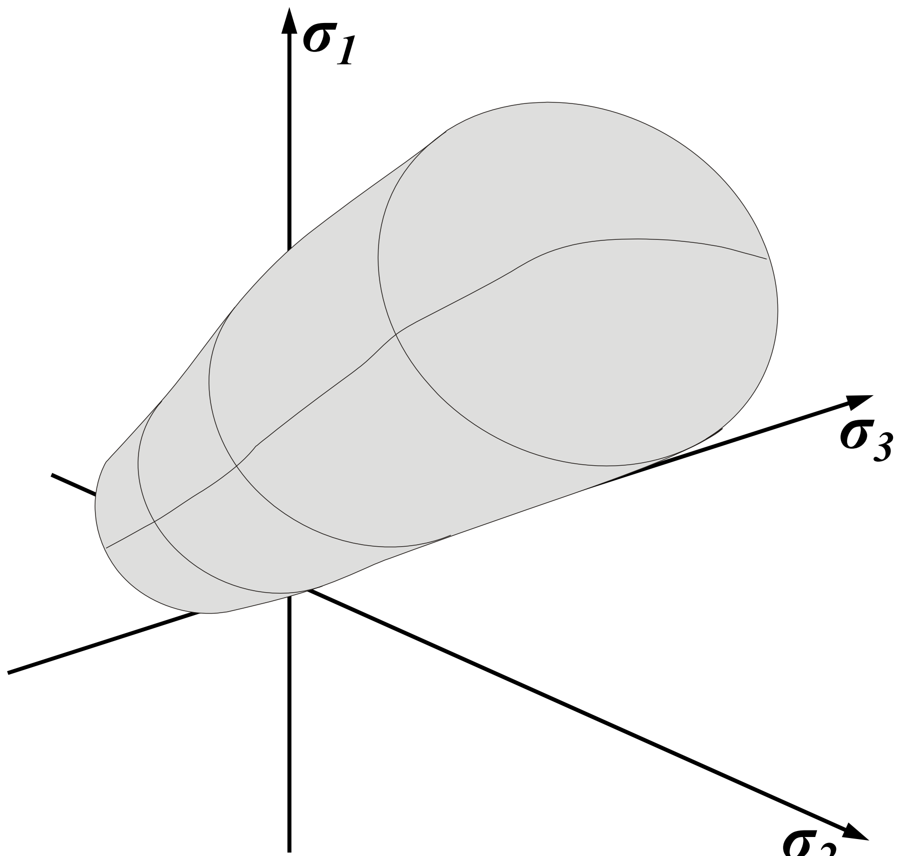
Failure process

1. Evidently, combinations of $\sigma_1, \sigma_2, \sigma_3$ exist such that the material does not fail for these values. (”Safe values”)
2. From experiments and experience we also know that combinations of $(\sigma_1, \sigma_2, \sigma_3)$ exist such that the material fails (breaks) at these values.

Theoretical experiment:
From a safe state, change stress values until material fails. Notice relevant $(\sigma_1, \sigma_2, \sigma_3)$.
Repeat experiment infinitely many times, each time arriving at a new triplet $(\sigma_1, \sigma_2, \sigma_3)$.

It is reasonable to assume that this collection of points will span a surface in $\boldsymbol{\sigma}$ – space: The failure surface.

Failure surface



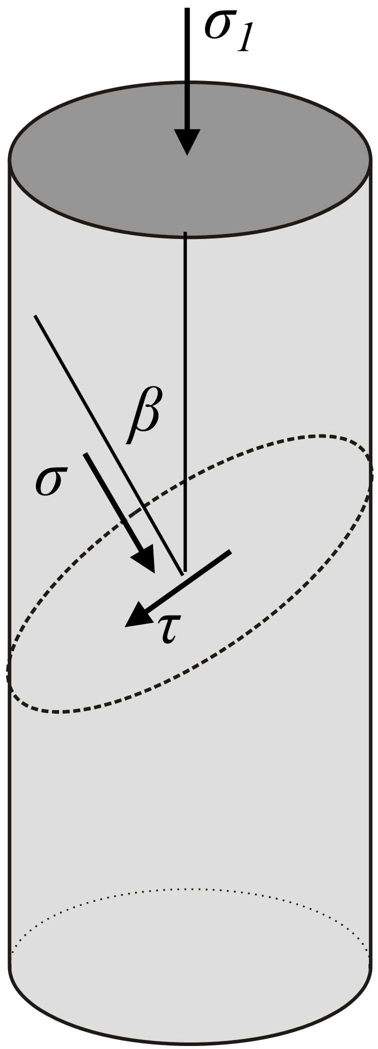
$$f(\sigma_1, \sigma_2, \sigma_3) = 0$$

All points interior to the surface are ”safe”.

Existence of failure surface implies that failure is independent of stress gradients and stress history, which may not be true.
But still good as a concept.

Note that our ”experiment” revealed one of the ”problems” of rock mech testing: It is impossible to repeat a failure experiment.

Mohr hypothesis



Shear failure occurs when shear stress τ along plane is too large. (σ is plane-normal stress.)

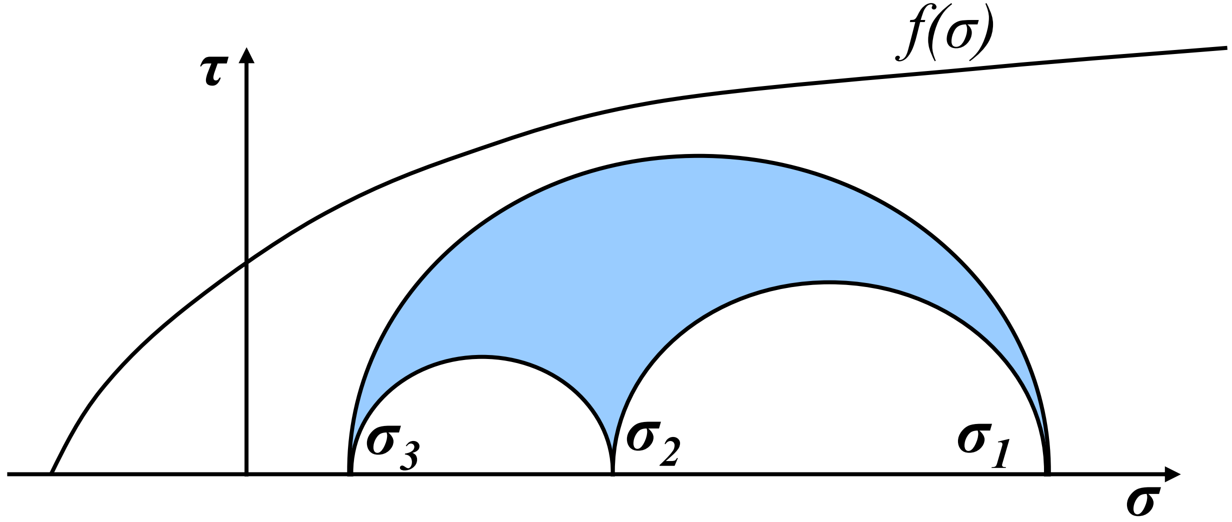
Mohr hypothesis: Failure can be described by

$$|\tau| = f(\sigma).$$

In the $(\tau - \sigma)$ - plane the equation describes some curve which separates a safe region from a failure region.

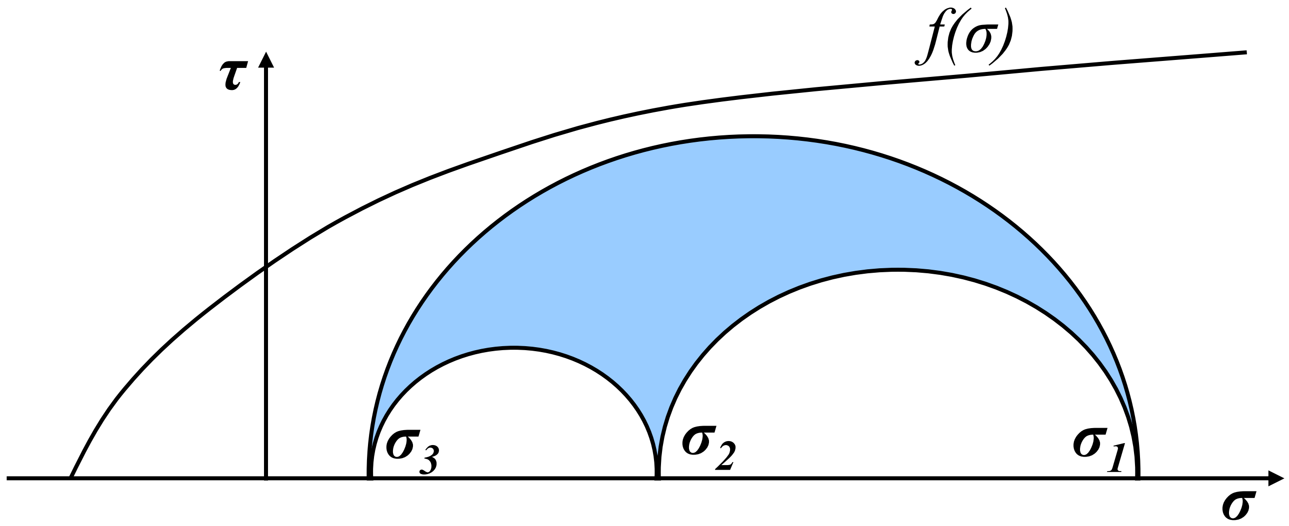
Note: Mohr hypothesis *only* applies to *shear failure*.

Shear failure

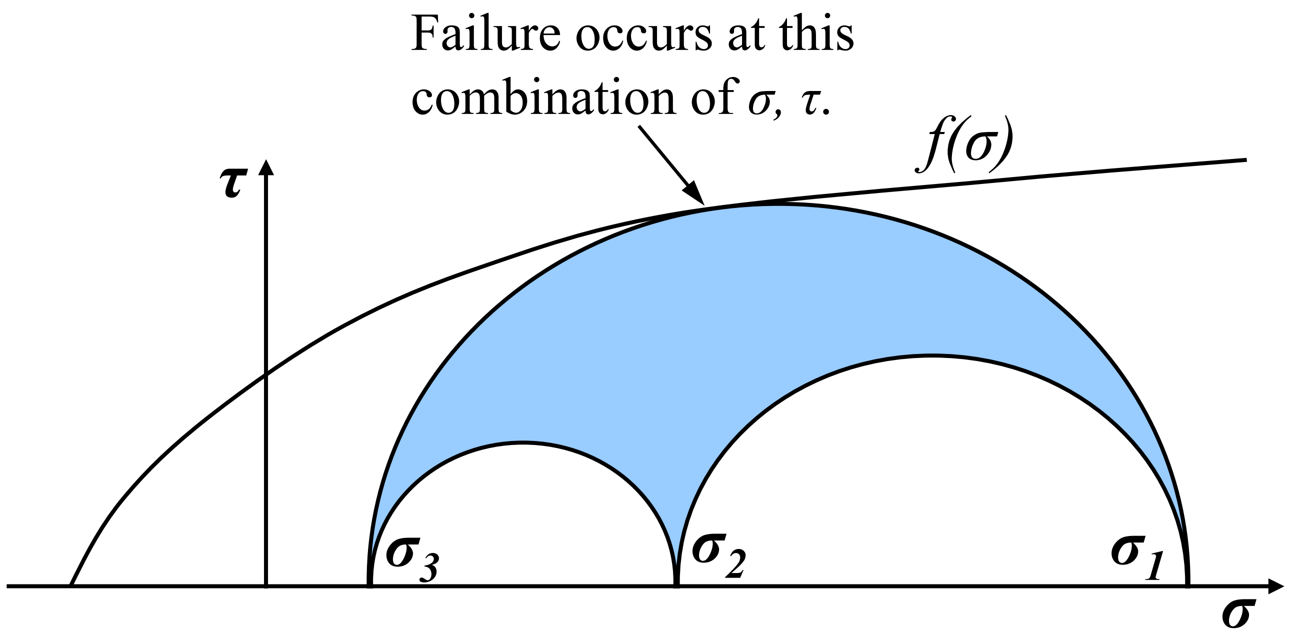


Recall the (closed) blue area contains all permitted combinations of *σ, τ*.
As drawn, the material cannot fail.

σ_1 is increased: Approaching failure



σ_1 is increased more: At failure

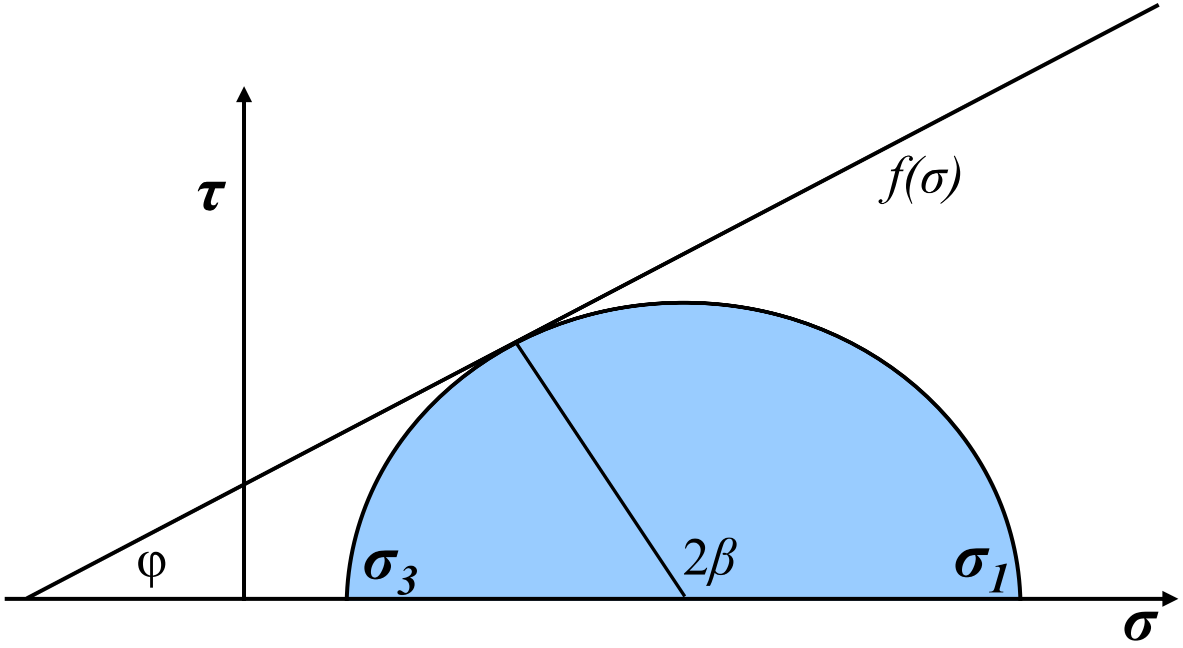


Note: At this stage we can't increase σ_1 further, because the failure encelope is the validity boundary for elastic behaviour.

Note2: Pure shear failure by Mohr hypothesis does not depend on intermediate stress.

The Mohr-Coulomb criterion

Specific choices of $f(\sigma)$ give different failure criteria.
Simplest: A straight line (MC)



$$|\tau| = S_0 + \mu\sigma$$

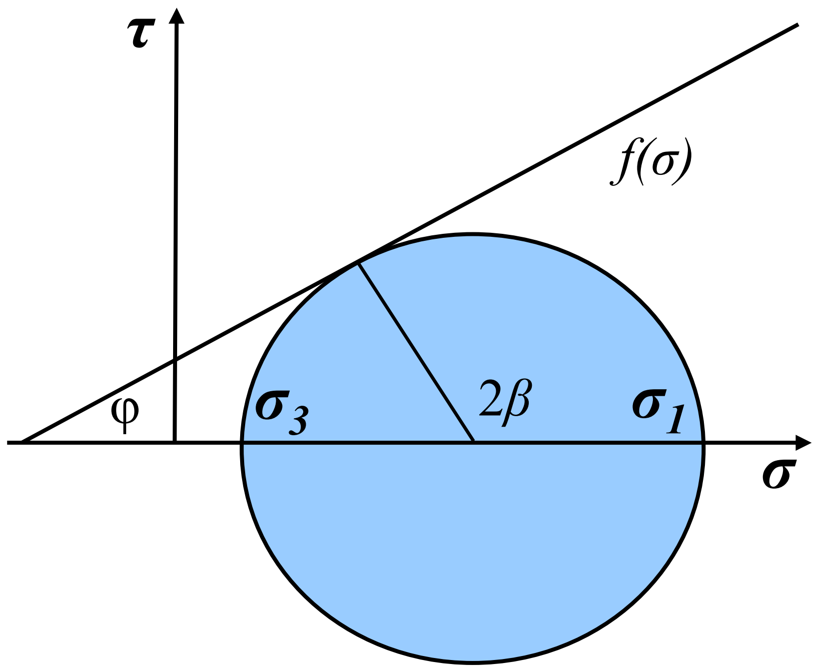
S_0: Inherent shear strength (Cohesion)

μ: Coeff. of internal friction

φ: Friction angle, $\tan\varphi = \mu$

Simple but very popular, and also successful, e.g. to describe fault failure

The Mohr-Coulomb criterion



Recall that β was the angle between the vertical and the failure plane normal

From the figure we see that

$$\varphi + \frac{\pi}{2} = 2\beta$$

or

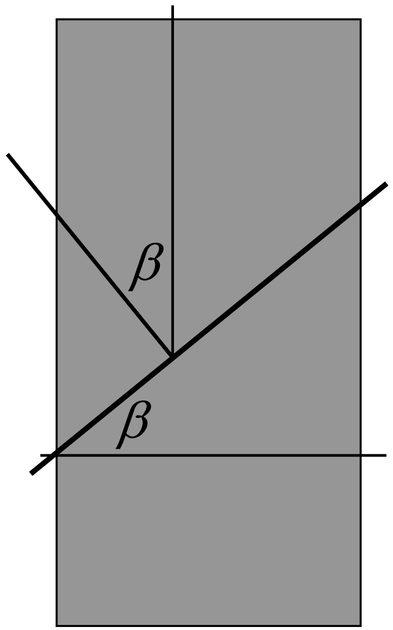
$$\beta = \frac{\pi}{4} + \frac{\varphi}{2}$$

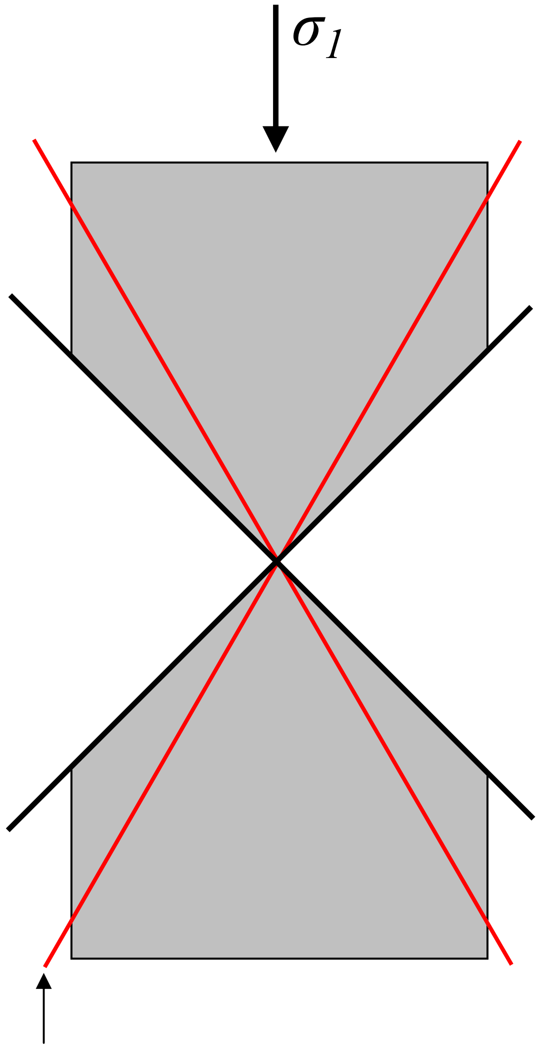
As $0 < \varphi < 90^\circ$, β must be in the range $45^\circ < \beta < 90^\circ$
(In practice φ is often not far from 30º, hence typically $\beta \approx 60^\circ$)

MC: Permitted failure planes

$45^o < \beta < 90^o$

often $\beta \approx 60^o$

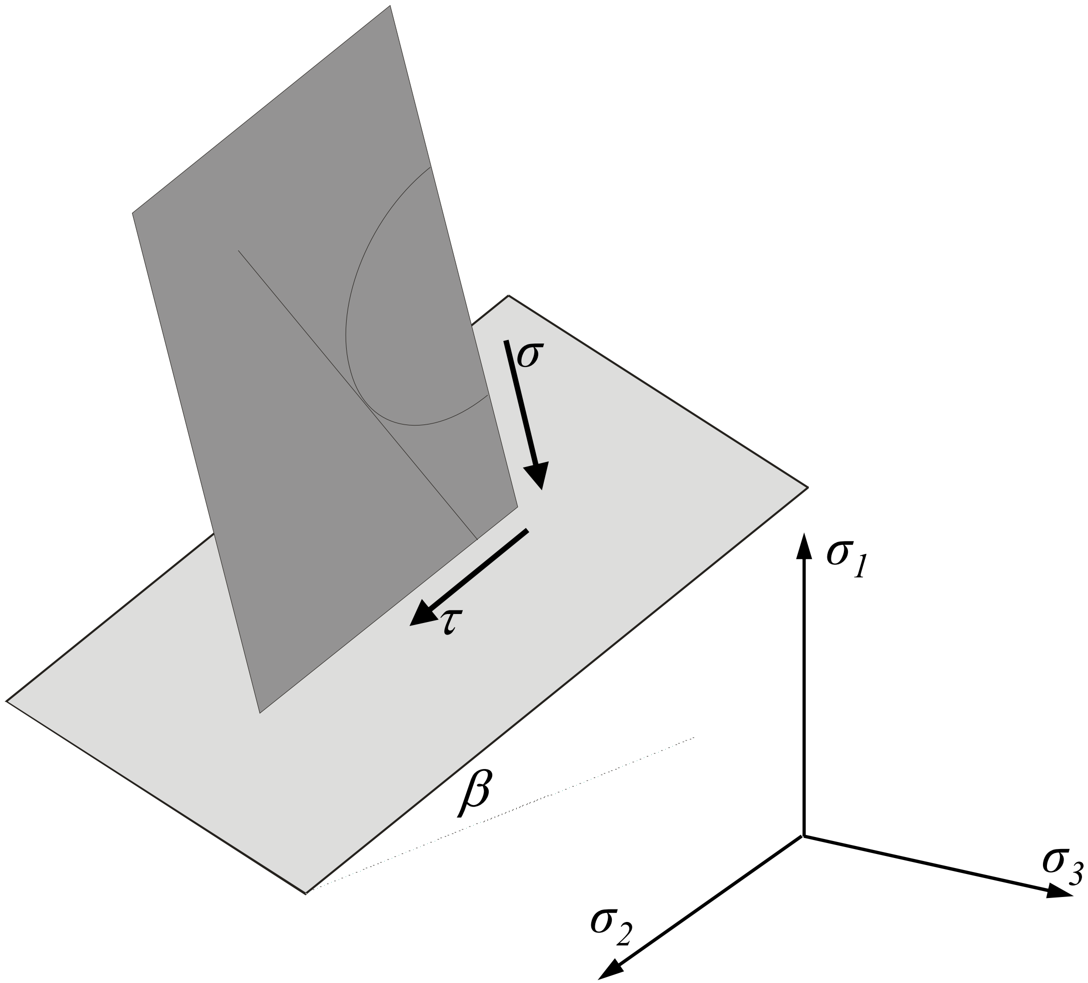




Not too uncommon fault plane slope direction

Note that by MC, the failure angle is determined solely by the friction angle, independent of the confining stress.

MC in 3-D stress space

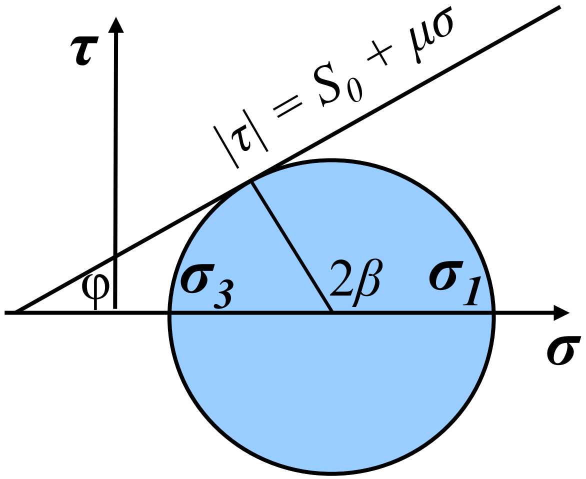


The Mohr-Coulomb criterion is analyzed in the (σ, τ)-plane (Normal to failure plane).

OK for a single failure plane, but we often want to operate in (fixed) **σ**-space.

The MC failure line will be a straight line for projections on all kinds of planes, but the slope angle will obviously vary.

MC in 3-D: Projection on (σ_1 - σ_3) - plane



From figure (and seminar 1), the tangent point is:

$$|\tau| = \frac{1}{2}(\sigma_1 - \sigma_3)\sin 2\beta$$

$$\sigma = \frac{1}{2}(\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3)\cos 2\beta$$

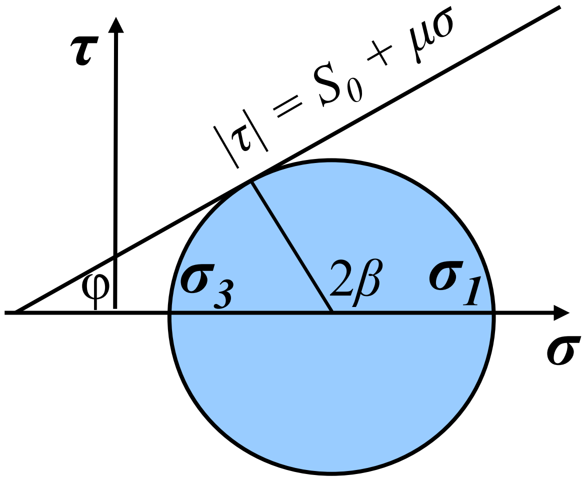
Using these values of (σ, τ) in the MC-equation:

$$\frac{1}{2}(\sigma_1 - \sigma_3)\sin 2\beta = S_0 + \mu\left[\frac{1}{2}(\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3)\cos 2\beta\right]$$

Replacing β and μ by φ (using prev. expressions) and manipulating:

$$\sigma_1 = 2S_0 \frac{\cos\varphi}{1 - \sin\varphi} + \sigma_3 \frac{1 + \sin\varphi}{1 - \sin\varphi}$$

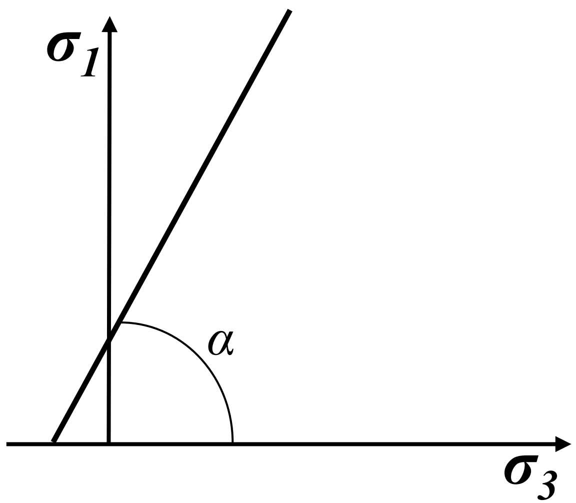
MC in 3-D: Projection on (σ_1 - σ_3) - plane



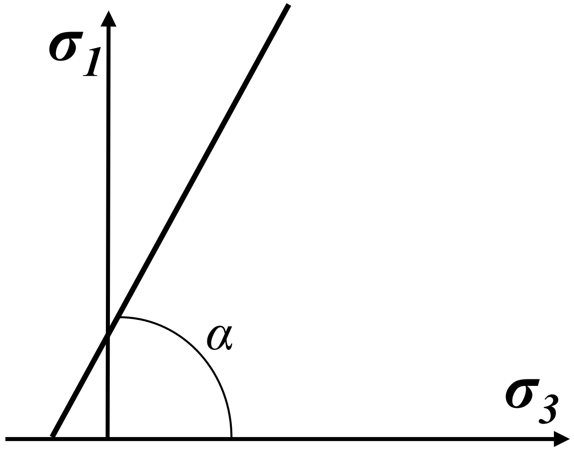
$$\sigma_1 = 2S_0 \frac{\cos\varphi}{1-\sin\varphi} + \sigma_3 \frac{1+\sin\varphi}{1-\sin\varphi}$$

Hence the projection of the MC-line on the (σ_1 - σ_3) – plane has a slope α:

$$\tan\alpha = \frac{1+\sin\varphi}{1-\sin\varphi}$$

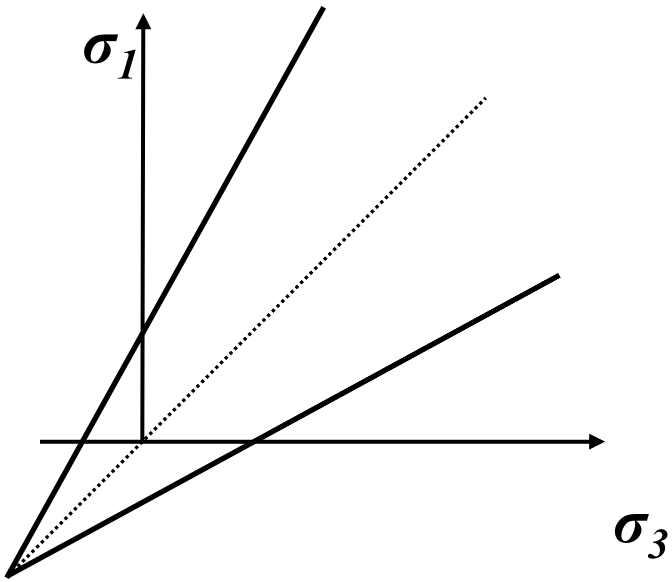


MC in 3-D: Projection on (σ_1 - σ_3) - plane



This line was from the assumption
$\sigma_1 > \sigma_2 > \sigma_3$.
The case $\sigma_3 > \sigma_2 > \sigma_1$ is symmetric about the line $\sigma_1 = \sigma_3$.

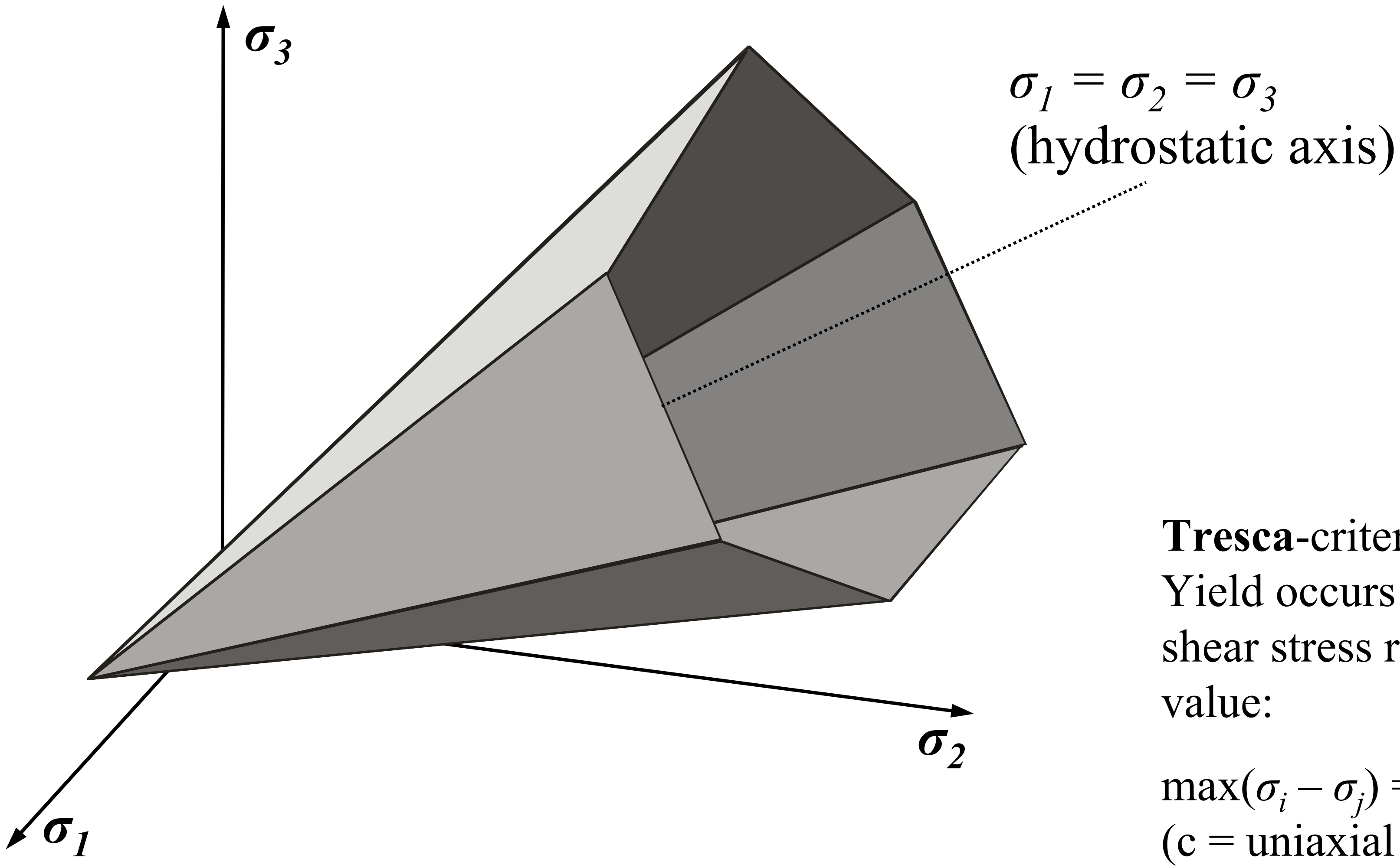
Both lines included in lower figure.



Projection (on (σ_1 - σ_3) – plane) of the part of the failure surface for which σ_2 is between σ_1 and σ_3.

Similar constructions can be done for the cases σ_1 intermidiate (on (σ_2 - σ_3) – plane) and σ_3 intermidiate.

MC failure surface in principal stress space



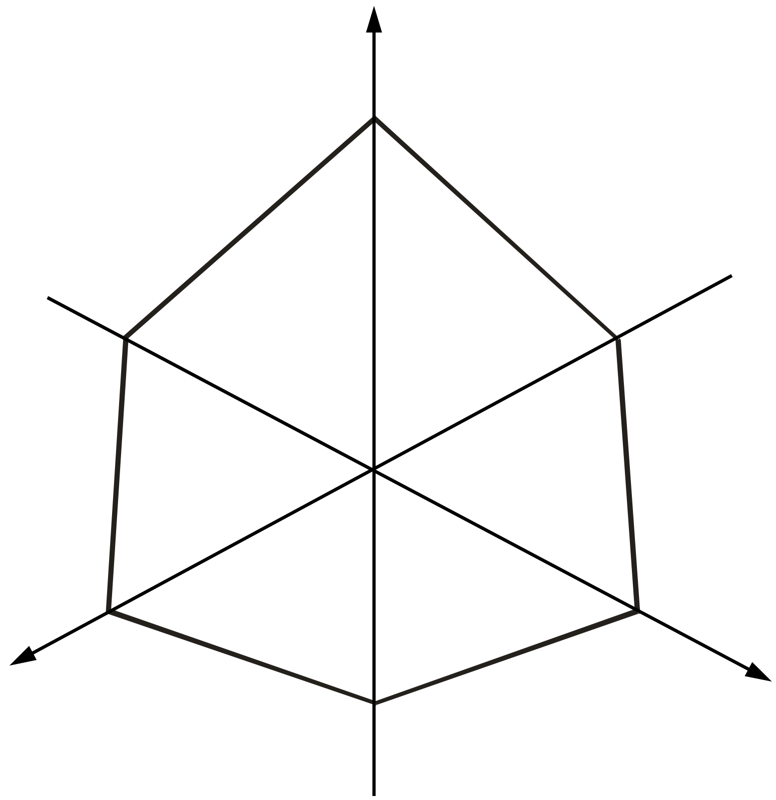
Tresca-criterion:
Yield occurs when max. shear stress reaches a crit. value:

$\max(\sigma_i - \sigma_j) = 2c,\ i,j = 1,2,3$
(c = uniaxial yield stress)

→ hexagonal prism

MC failure surface in π-plane

3-D drawings can be difficult to make and analyze. Failure surfaces are often studied in a plane, of which a much used is the π-plane, which is normal to the hydrostatic axis.



Mohr Coulomb summary

- ▲ Simple – easy to analyze
- ▲ Reasonably good predictive ability (e.g. fault slopes)
- ▼ Independent of intermediate stress (not always true)
- ▼ Non-smooth failure surface (physics? Numerics?)

Other similar criteria

A number of failure criteria has been constructed by either assuming another form of $f(\sigma)$, or by direct assumptions on the shape of the failure surface.
All of these attempt to improve match to experiments.

Some mentioned:

- Tresca (prism in lieu of pyramid)
- von Mises (smooth version of Tresca (elliptic prism))
- Griffith
- Drucker-Prager
- +++

If interested consult relevant litterature.

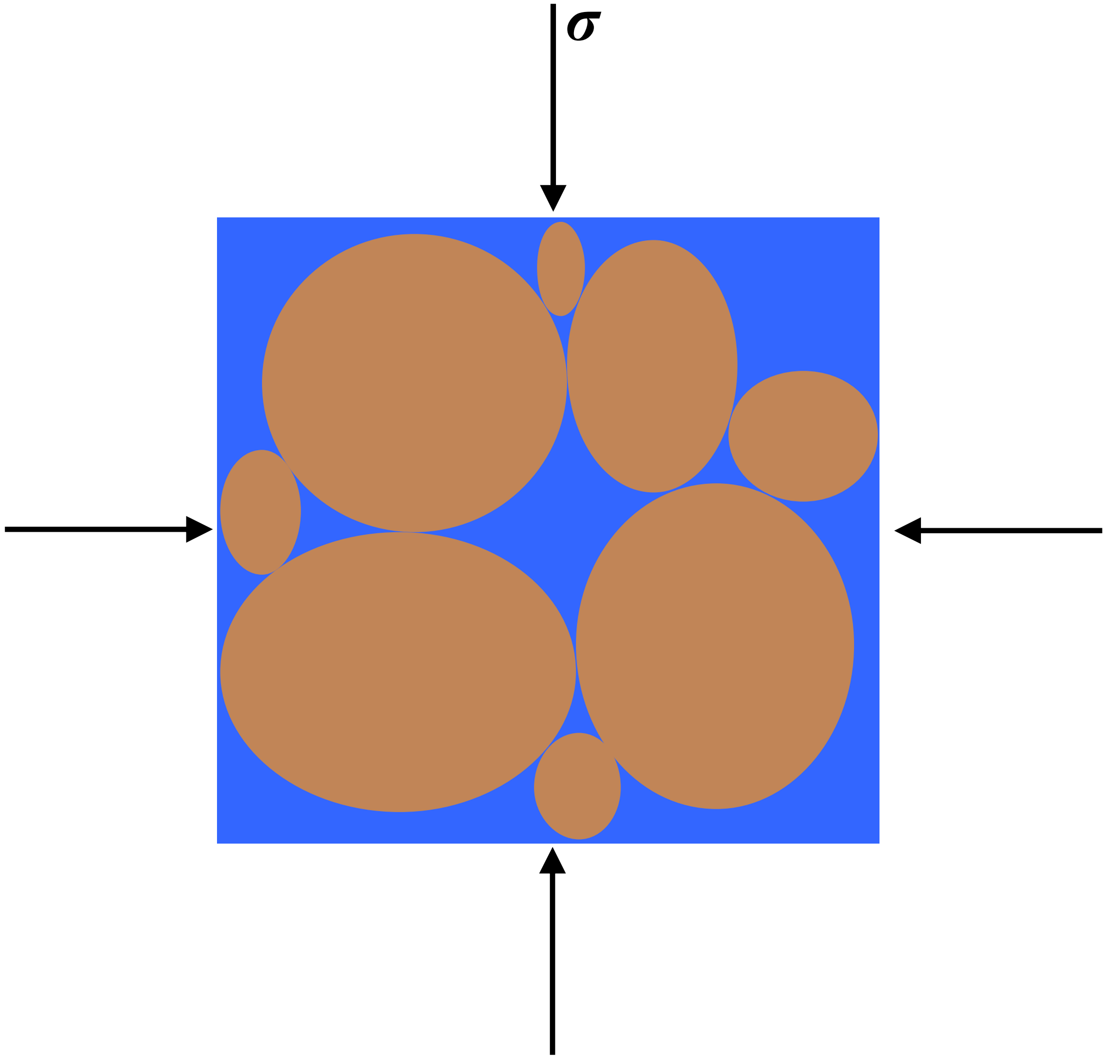
Failure under tension

Tension is expansion, i.e. in a porous rock, the pore pressure will be increased to a level such that the effective stress becomes negative, and lower than the material's tensile strength.

Typical examples are fracturing near an injection well, and (pehaps) overburden fracturing if reservoir pressure is sufficiently increased (”balloon effect”).

The theory is complex – too complex for this seminar. So let's leave it with that…

Grain packing and failure



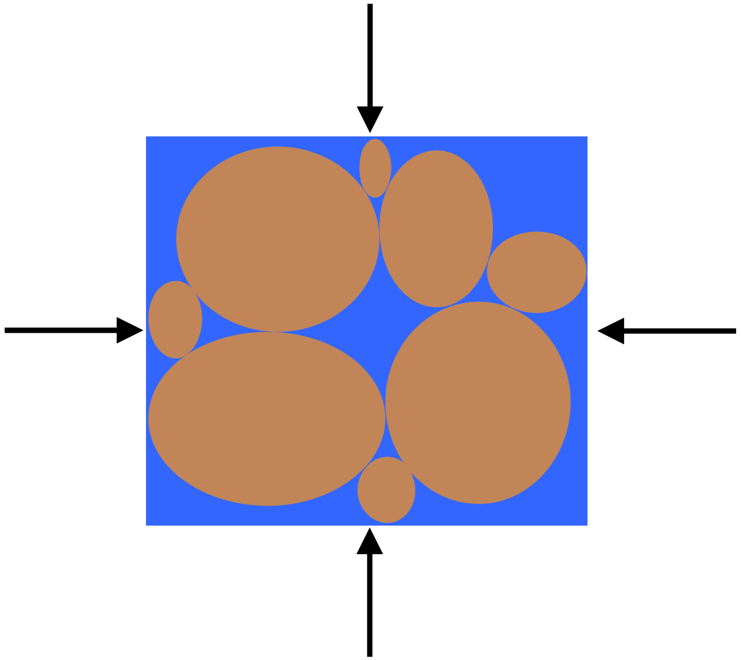
Consider a granular sample under compression.

Typical compression coeff's:
Bulk: $(0.1 – 1\ \text{GPa})^{-1}$ (sand)
$(5 – 15\ \text{GPa})^{-1}$ (sandstone)
Grains: $\approx(38\ \text{GPa})^{-1}$ (quartz)

I.e. Bulk compressibility is much larger than grain compressibility; for sands 2-3 orders of magnitude larger.

Grain compression is insignificant compared to pore volume reduction

Grain packing and failure



→(As good as) The entire compaction must be attributed to pore volume reduction.

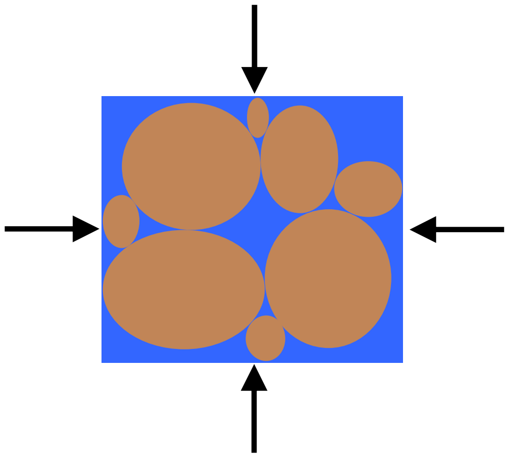
Key observation:
If grains are (almost) incompressible, it is impossible to compress the sample without reorganizing the grains. (Try it!)

Hence,
Each level of compaction corresponds to some grain packing configuration.

During compaction, the pore walls are continuously failing. (As far from elasticity as we can get (?))

Principle of stable settlement

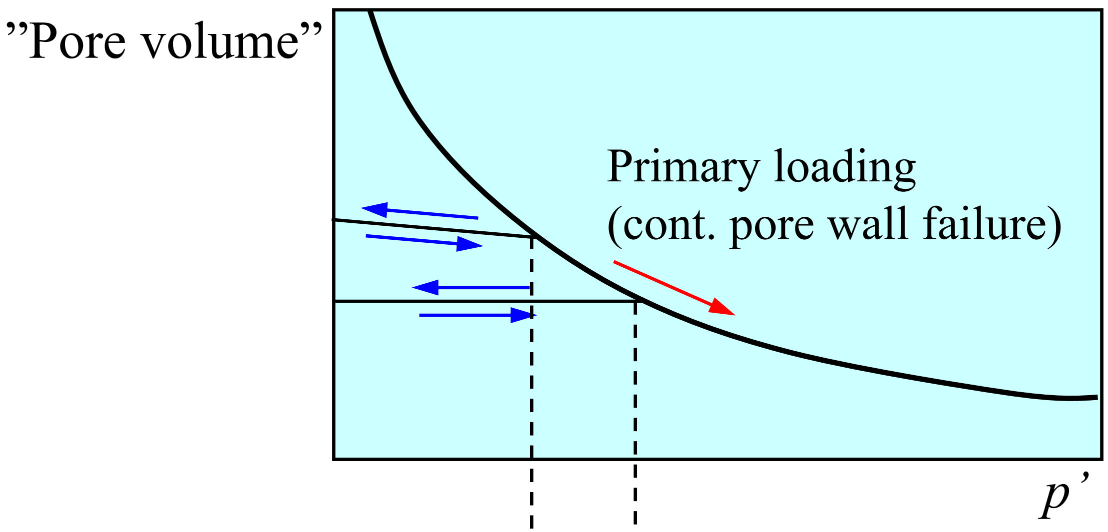
- When grains reorganize they will always tend to seek the most stable packing pattern available.
- Grains will never reconfigure from the current pattern to a less stable one.

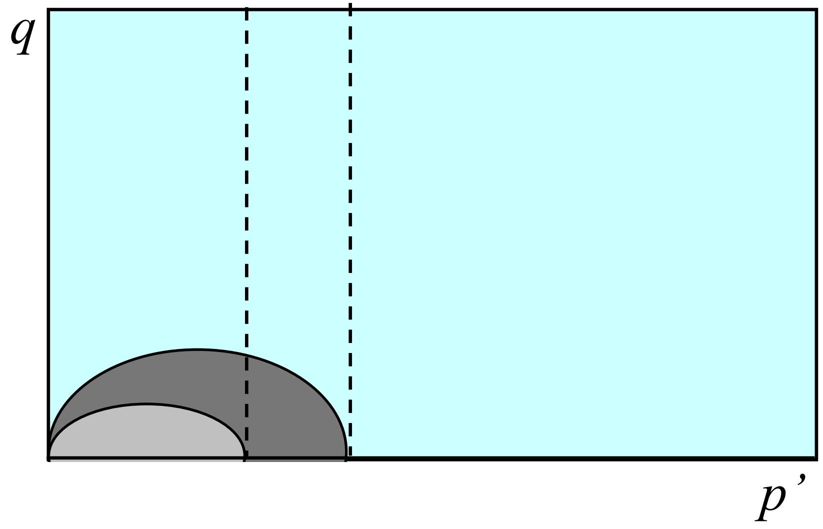


Consequences of stable settlement

- In a loading process, each (eff.) stress state corresponds to a stable packing
  - the tightest possible packing at that stress level.
- The soil has no memory of its previous states (solids do!)
  - Each packing level can be regarded as a ”new” material with its own poro-elasto-plastic parameters
- Packing will be increasingly harder to achieve as it becomes tighter.
  - Each ”packing level” is more stable than previous levels
  - Bulk modulus increases with effective stress
- Relieving stress will not return the soil to a previous (less stable) level
  - Permanent deformation
  - Present packing is a result of the historical maximal effective stress

Intuitive qualitative soil response





Primary reduction of pore volume requires increasingly larger (eff.) stress increase

→ Material hardens

Blue arrows:

Unloading – reloading:

No or small volume change.

≈ linear elastic behaviour

In stress-space (now using *p’* and *q*), by analogy with MC, we have a ”safe” region, where material behaves elastic (grey). Unloading – reloading is within this region.

To move down the primary load line, we must increase the size of the ”current region”. How do we do that?

Open questions

What does the elastic region boundary look like?

How does it change when the material changes from elastic to plastic?

How does the material harden / soften?

What are the consequences of hardening / softening (for instance w.r.t. failure) ?

Tune in on the following seminar(s)!